A BLOCK FACTOR ANALYSIS BASED RECEIVER FOR BLIND MULTI-USER ACCESS IN WIRELESS COMMUNICATIONS

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ABSTRACT

In this paper, we present a technique for the blind separation of DS-CDMA signals received on an antenna array, for a multi-path propagation scenario with Inter Symbol Interference. Our method relies on a new third-order tensor decomposition, which is a generalization of the parallel factor model. We start with the observation that the temporal, spatial and spectral diversities give a third-order tensor structure to the received data. This tensor is then decomposed in a sum of contributions by means of an Alternating Least Squares algorithm, where each contribution fully characterizes one user.

1. INTRODUCTION

Let us consider R users transmitting frames of J symbols at the same time within the same bandwidth towards an array of K antennas. We denote by I the spreading factor, i.e. the CDMA code of each user is a vector of length I. In a direct-path only propagation scenario, the assumption that the channel is noiseless / memoryless leads to the following data model:

$$y_{ijk} = \sum_{r=1}^{R} h_{ir} s_{jr} a_{kr}, \tag{1}$$

where y_{ijk} is the output of the k^{th} antenna for chip i and symbol j. The scalar a_{kr} is the gain between user r and antenna element k, s_{jr} is the j^{th} symbol transmitted by user r and h_{ir}, for varying i and fixed r, is the result of convolving the spreading sequence of user r with the impulse response of its propagation channel. For background material on algebraic solutions to this problem, we refer to [1, 2]. In this article, we focus on a more complex situation where multi-path propagation leads to Inter-Symbol-Interference (ISI). We also assume that the reflections can both occur in the far and close fields of the antenna array so that each path is characterized by its own delay τ_p , angle of arrival θ_p and attenuation α_p, where p denotes the path index. Under these assumptions, our objective is to estimate the symbols transmitted by every user in a blind way, without using any prior knowledge on the propagation parameters. Our approach consists of collecting the received data in a third-order tensor and to express this tensor as a sum of R contributions by means of a new tensor decomposition. In section 2 we will generalize the model (1) to the propagation scenario that is under consideration. In section 3 we will introduce some multilinear algebra prerequisites. In section 4, our new tensor decomposition is applied to the data model introduced in section 2. In section 5 we propose an algorithm to compute this decomposition and we analyze its performance.

2. DATA MODEL

Let us start with a single source transmitting J symbols along P paths towards K antennas. These paths can be considered as channels with memory, leading to ISI, and are assumed to be stationary over J symbols. Let L be the maximum channel length at the symbol rate, meaning that interference is occurring over L symbols. The coefficients resulting from the convolution between the channel impulse response for the p^{th} path and the spreading sequence of the user under consideration are collected in a vector h_p of size LI. So $h_p(i+(l-1)I)$ is the coefficient of the overall impulse response corresponding to the i^{th} chip and the l^{th} symbol. We denote by $x_p(i,j)$ the i^{th} chip of the signal received from the p^{th} path during the j^{th} symbol period. We have:

$$x_p(i,j) = \sum_{l=1}^{L} h_p(i + (l-1)I)\, s_{j-l+1} \tag{2}$$

Let $a_k(\theta_p)$ be the response of the k^{th} antenna to the signal coming from the p^{th} path with an angle of arrival θ_p, where we assume that the path loss is combined with the antenna gain. The model defined in (2) then yields:

$$x_p(i,j,k) = a_k(\theta_p) \sum_{l=1}^{L} h_p(i + (l-1)I)\, s_{j-l+1}, \tag{3}$$

where $x_p(i,j,k)$ denotes the i^{th} chip of the j^{th} symbol of the signal received by the k^{th} antenna. We now write the overall received signal by summing the contributions of the P paths and the R users:

$$y_{ijk} = \sum_{r=1}^{R} \sum_{p=1}^{P} a_k(\theta_{rp}) \sum_{l=1}^{L} h_{rp}(i + (l-1)I)\, s_{j-l+1}^{(r)}, \quad (4)$$

where y_{ijk} denotes the i^{th} chip of the j^{th} symbol of the signal received by the k^{th} antenna, and in which r, p and l are the user, path and interfering symbol index respectively.

3. MULTILINEAR ALGEBRA PREREQUISITES

A quantity of which the elements are addressed by N indices is an Nth-order tensor or N-way array. Signal processing based on multilinear algebra is discussed in [3].

Definition 1. (Mode-n product) *The mode-1 product of a third-order tensor $\mathcal{Y} \in \mathbb{C}^{L\times M\times N}$ by a matrix $\mathbf{A} \in \mathbb{C}^{I\times L}$, denoted by $\mathcal{Y} \times_1 \mathbf{A}$, is an $(I \times M \times N)$-tensor with elements defined, for all index values, by*

$$(\mathcal{Y} \times_1 \mathbf{A})_{imn} = \sum_{l=1}^{L} y_{lmn} a_{il}$$

Similarly, the mode-2 product by a matrix $\mathbf{B} \in \mathbb{C}^{J\times M}$ and the mode-3 product by $\mathbf{C} \in \mathbb{C}^{K\times N}$ are the $(L \times J \times N)$ and $(L \times M \times K)$ tensors respectively, with elements defined by

$$(\mathcal{Y} \times_2 \mathbf{B})_{ljn} = \sum_{m=1}^{M} y_{lmn} b_{jm}$$

$$(\mathcal{Y} \times_3 \mathbf{C})_{lmk} = \sum_{n=1}^{N} y_{lmn} c_{kn}$$

In this notation, the matrix product $\mathbf{Y} = \mathbf{U}.\mathbf{S}.\mathbf{V}^T$ takes the form of $\mathbf{Y} = \mathbf{S} \times_1 \mathbf{U} \times_2 \mathbf{V}$.

Definition 2. (Rank-1 Tensor) *$\mathcal{Y} \in \mathbb{R}^{I\times J\times K}$ is of rank-1 if its elements can be written as $y_{ijk} = \mathbf{a}(i)\mathbf{b}(j)\mathbf{c}(k)$, where $\mathbf{a} \in \mathbb{C}^{I\times 1}$, $\mathbf{b} \in \mathbb{C}^{J\times 1}$ and $\mathbf{c} \in \mathbb{C}^{K\times 1}$.*

This definition generalizes the definition of a rank-1 matrix: $\mathbf{A} \in \mathbb{C}^{I\times J}$ has rank 1 if $\mathbf{A} = \mathbf{a}.\mathbf{b}^T$.

Definition 3. (Tensor Rank) *The rank of $\mathcal{Y}$ is defined as the minimum number of rank-1 tensors that yield $\mathcal{Y}$ in a linear combination.*

4. TENSOR DECOMPOSITIONS

4.1. PARAFAC Decomposition

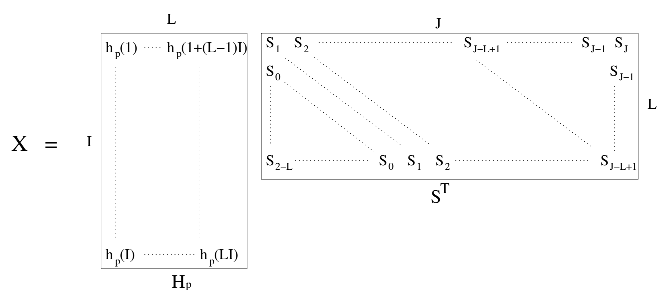
Parallel Factor Analysis (PARAFAC) was introduced by Harshman in [4]. It is a powerful technique to decompose a rank-R tensor in a linear combination of R rank-1 tensors. Let $\mathcal{Y}$ be an $(I \times J \times K)$ tensor, with elements denoted by y_{ijk}. The PARAFAC decomposition of $\mathcal{Y}$ can be written as

$$y_{ijk} = \sum_{r=1}^{R} \mathbf{a}_r(i)\mathbf{b}_r(j)\mathbf{c}_r(k), \quad (5)$$

where $\mathbf{a}_r$, $\mathbf{b}_r$, $\mathbf{c}_r$ are the r^{th} columns of matrices $\mathbf{A} \in \mathbb{C}^{I\times R}$, $\mathbf{B} \in \mathbb{C}^{J\times R}$ and $\mathbf{C} \in \mathbb{C}^{K\times R}$ respectively, and where i, j and k denote the row index. It now appears that the model for a memoryless channel (1) can be seen as a PARAFAC decomposition of the observation tensor $\mathcal{Y}$. Sidiropoulos was the first to use this multilinear algebra technique in the context of wireless communications [2]. The model that takes into account multi-path and ISI (4) can be seen as a tensor decomposition that is more general than PARAFAC. This technique is a special case of Block Factor Analysis (BFA) [5].

4.2. Block Factor Analysis

We start with Eq. (2), in which $x_p(i, j)$ is considered as an element of an $I \times J$ matrix $\mathbf{X}$, resulting from the product of an $I \times L$ matrix $\mathbf{H}_p$ and an $L \times J$ Toeplitz matrix $\mathbf{S}^T$:



After incorporating the antenna array response we obtain the tensor model for (3):

$$\mathcal{X}_p = \mathbf{H}_p \times_2 \mathbf{S} \times_3 \mathbf{a}_p, \quad (6)$$

where $\mathcal{X}_p$ is an $(I \times J \times K)$ tensor that represents the contribution of the p^{th} path from a single user, and $\mathbf{a}_p$ is a $K \times 1$ vector that contains the antenna array response to the p^{th} path. Considering all P paths, the overall signal associated with the user under consideration is given by:

$$\mathcal{X} = \sum_{p=1}^{P} \mathcal{X}_p = \sum_{p=1}^{P} \mathbf{H}_p \times_2 \mathbf{S} \times_3 \mathbf{a}_p \quad (7)$$

This equation can be rewritten in a more compact way:

$$\mathcal{X} = \mathcal{H} \times_2 \mathbf{S} \times_3 \mathbf{A}, \quad (8)$$

where $\mathcal{H}$ is an $I \times L \times P$ tensor with each frontal slice equal to one of the $\mathbf{H}_p$ matrices, $\mathbf{S}$ is the $J \times L$ Toeplitz source matrix and $\mathbf{A}$ the $K \times P$ matrix that contains the set of vectors

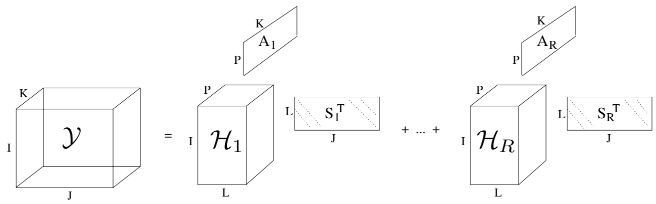


Fig. 1. Schematic representation of the BFM

$\mathbf{a}_p$. Finally, we consider R users transmitting at the same time, each along P paths, so we obtain the following tensor equivalent of (4):

$$\mathcal{Y} = \sum_{r=1}^{R} \mathcal{H}_r \times_2 \mathbf{S}_r \times_3 \mathbf{A}_r \tag{9}$$

This Block Factor Model (BFM) is represented in Figure 1. Each term of the sum contains the information related to one particular user: the global channel is characterized by the tensor $\mathcal{H}_r$, the antenna array response is given by $\mathbf{A}_r$ and the J transmitted symbols are collected in $\mathbf{S}_r$.

4.3. Uniqueness of the GCD

If the BFM (9) is unique (up to some trivial indeterminacies), then its computation allows for the separation of the different user signals and the estimation of the transmitted sequences. We call a property generic when it holds everywhere, except for a set of Lebesgue measure 0. A generic condition for uniqueness has been derived in [5]:

$$min\left(\left\lfloor \frac{J}{L} \right\rfloor, R\right) + min\left(\left\lfloor \frac{K}{P} \right\rfloor, R\right) + min\left(\left\lfloor \frac{I}{max(L,P)} \right\rfloor, R\right) \geq 2R+2, \tag{10}$$

if $I > L + P - 2$. If $I \leq L + P - 2$, then some additional conditions apply. This result implies an upper bound on the number of users that can be allowed at the same time. The maximal number of simultaneous users correspond to the maximal value R that satisfies (10).

5. COMPUTATION OF THE GCD

5.1. Algorithm

Given only $\mathcal{Y}$, we want to estimate $\mathcal{H}_r$, $\mathbf{S}_r$ and $\mathbf{A}_r$ for each user. We present an Alternating Least Squares algorithm (ALS), consisting of alternating conditional updates of these unknowns. We denote by $\mathbf{A}$ and $\mathbf{S}$ the $K \times RP$ and $J \times RL$ matrices that result from the concatenation of the R matrices $\mathbf{A}_r$ and $\mathbf{S}_r$ respectively. We start the algorithm with random initialization of all $\mathcal{H}_r$ and $\mathbf{S}_r$**.**

1. First Step: Update of $\mathbf{A}$.

We suppose $\mathbf{S}$ and each $\mathcal{H}_r$ known from previous iteration. Consider the $JI \times K$ matrix representation of $\mathcal{Y}$, defined by $[\mathbf{Y}^{(JI \times K)}]_{(j-1)J+i,k} = y_{ijk}$. This matrix can be considered as the result of row-wise concatenation of the J left-right slices of $\mathcal{Y}$. Note the order in which the entries are stacked, with the left index (j here) varying more slowly than the right one. We will adopt the same ordering convention below. We then write (9) as $\mathcal{Y} = \sum_{r=1}^{R} \mathcal{Q}_r \times_3 \mathbf{A}_r$ where we define $\mathcal{Q}_r = \mathcal{H}_r \times_2 \mathbf{S}_r$ as an $(I \times J \times P)$ tensor. In matrix format we get $\mathbf{Y}^{(JI \times K)} = \sum_{r=1}^{R} \mathbf{Q}_r^{(JI \times P)} \mathbf{A}_r^T$, such that (9) can be rewritten as :

$$\mathbf{Y}^{(JI \times K)} = \mathbf{Q}.\mathbf{A}^T, \tag{11}$$

where $\mathbf{Q}$ is the $JI \times RP$ matrix resulting from the concatenation of the matrices $\mathbf{Q}_r^{(JI \times P)}$. Finally we obtain an expression for the least squares update of $\mathbf{A}$:

$$\hat{\mathbf{A}} = (\mathbf{Y}^{(JI \times K)})^T (\mathbf{Q}^T)^{\dagger}, \tag{12}$$

where $\dagger$ denotes the pseudo-inverse.

2. Second Step: Update of $\mathbf{S}$.

We suppose $\mathbf{A}$ and each $\mathcal{H}_r$ known. In order to preserve the Toeplitz structure, we will update the generator vector of each $\mathbf{S}_r$ instead of $\mathbf{S}_r$ itself. Equation (9) can also be written as $\mathcal{Y} = \sum_{r=1}^{R} \mathcal{G}_r \times_2 \mathbf{S}_r$, where $\mathcal{G}_r = \mathcal{H}_r \times_3 \mathbf{A}_r$. Let $\mathbf{s}_r$ be the $(J + L - 1) \times 1$ generator vector of the Toeplitz matrix corresponding to the r^{th} user: $\mathbf{s}_r = [s_{2-L} \dots s_1 \dots s_J]^T$. Front-back slices of $\mathcal{Y}$ and $\mathcal{G}_r$ are $I \times J$ and $I \times L$ matrices obtained by keeping index k fixed and are respectively written as $\mathbf{Y}_k$ and $\mathbf{G}_{r,k}$. We build the following $JI \times (J+L-1)$ matrix $\mathbf{M}_{r,k}$ from $\mathbf{G}_{r,k}$:

$$\mathrm{M}_{r,k} = \begin{pmatrix} [\mathrm{G}_{rk}]_{:,L} & \dots & [\mathrm{G}_{rk}]_{:,1} & 0_I & \dots & \dots & \dots & \dots & \dots & 0_I \\ 0_I & [\mathrm{G}_{rk}]_{:,L} & \dots & [\mathrm{G}_{rk}]_{:,1} & 0_I & \dots & \dots & \dots & \dots & 0_I \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0_I & \dots & \dots & \dots & \dots & \dots & 0_I & [\mathrm{G}_{rk}]_{:,L} & \dots & [\mathrm{G}_{rk}]_{:,1} \end{pmatrix}$$

where $[\mathbf{G}_{r,k}]_{:,l}$ denotes the l^{th} column of $\mathbf{G}_{r,k}$ and $\mathbf{0}_I$ is an $I \times 1$ vector of zeros.

Let us denote by Vec the operator that writes a matrix $\mathbf{A} \in \mathbb{C}^{I \times R}$ in vector format by concatenation of the columns such that $\mathbf{A}(i,r) = [\mathrm{Vec}(\mathbf{A})]_{i+(r-1)I}$. For index k fixed, we get: $\mathrm{Vec}(\mathbf{Y}_k) = \sum_{r=1}^{R} \mathbf{M}_{r,k}\mathbf{s}_r$, with $\mathrm{Vec}(\mathbf{Y}_k)$ of size $JI \times 1$. Taking into account all k, we obtain:

$$Y^{(KJI \times 1)} = \sum_{r=1}^{R} \mathbf{M}_r \mathbf{s}_r, \tag{13}$$

where $\mathbf{M}_r$ results from row-wise concatenation of all $\mathbf{M}_{r,k}$ and $Y^{(KJI \times 1)}$ from row-wise concatenation of all $\mathrm{Vec}(\mathbf{Y}_k)$.

Equation (13) can itself be written as a single matrix multiplication:

$$Y^{(KJI \times 1)} = \mathbf{M}\,\mathbf{s}, \tag{14}$$

where $\mathbf{M}$ is a $KJI \times R(J+L-1)$ matrix obtained by column wise concatenation of all $\mathbf{M}_r$ and $\mathbf{s}$ is an $R(J+L-1) \times 1$ vector resulting from row-wise concatenation of the R generator vectors. A least squares update of $\mathbf{s}$ can be computed from (14). Then we rebuild each $\mathbf{S}_r$ from the generator vectors.

3. Third Step: Update of each $\mathcal{H}_r$

Now we suppose $\mathbf{A}$ and $\mathbf{S}$ known. Top-bottom slices of $\mathcal{Y}$ and $\mathcal{H}_r$, i.e. $K \times J$ and $P \times L$ matrices obtained by keeping index i fixed, are written as $\mathbf{Y}_i$ and $\mathbf{H}_{r,i}$ respectively. For index i fixed, (9) leads to: $\text{Vec}(\mathbf{Y}_i) = \sum_{r=1}^{R} (\mathbf{S}_r \otimes \mathbf{A}_r) \text{Vec}(\mathbf{H}_{r,i})$, in which $\otimes$ denotes the Kronecker product. Now, $\text{Vec}(\mathbf{Y}_i)$ and $\text{Vec}(\mathbf{H}_{r,i})$ can be interpreted as the i^{th} column vectors of matrices $\mathbf{Y}^{(JK \times I)}$ and $\mathbf{H}_r^{(LP \times I)}$ respectively, so that we obtain $\mathbf{Y}^{(JK \times I)} = \sum_{r=1}^{R} (\mathbf{S}_r \otimes \mathbf{A}_r) \mathbf{H}_r^{(LP \times I)}$ from the previous equation. Finally, (9) can be written as:

$$\mathbf{Y}^{(JK \times I)} = \mathbf{Z}\mathbf{H}^{(RLP \times I)}, \tag{15}$$

where $\mathbf{Z} = [\mathbf{S}_1 \otimes \mathbf{A}_1, ..., \mathbf{S}_R \otimes \mathbf{A}_R]$ is a $JK \times RLP$ matrix and $\mathbf{H}^{(RLP \times I)}$ results from the concatenation of all $\mathbf{H}_r^{(LP \times I)}$. Eq. (15) gives the least squares update of $\mathbf{H}^{(RLP \times I)}$. Then the entries of $\hat{\mathbf{H}}^{(RLP \times I)}$ are stacked in R tensors $\mathcal{H}_r$.

We finally build an ALS-type algorithm for the computation of the BFM (9) from these three update rules. We denote by $\mathcal{Y}^{(n)}$ the tensor built at the n^{th} iteration from the R updated factors $\mathcal{H}_r^{(n)}$, $\hat{\mathbf{S}}_r^{(n)}$ and $\hat{\mathbf{A}}_r^{(n)}$ and we define the following function: $c(n) = \|\mathcal{Y}^{(n)} - \mathcal{Y}^{(n-1)}\|$.

Summary of the algorithm:

1- Initialize randomly each $\mathcal{H}_r$ and $\mathbf{S}_r$

2- Update $\mathbf{A}$ from (12): $\hat{\mathbf{A}} = (\mathbf{Y}^{(JI \times K)})^T (\mathbf{Q}^T)^\dagger$

3- Update $\mathbf{S}$ from (14): $\hat{\mathbf{s}} = \mathbf{M}^\dagger Y^{(KJI \times 1)}$

4- Update $\mathcal{H}_r$ from (15): $\hat{\mathbf{H}}^{(RLP \times I)} = (\mathbf{Z})^\dagger \mathbf{Y}^{(JK \times I)}$

5- Repeat from 2 until $c(n) < \epsilon$ (e.g. $\epsilon = 10^{-5}$)

5.2. Results of simulations

We illustrate the performance of our algorithm in presence of additive white Gaussian noise, so that equation (9) becomes: $\mathcal{Y}_{obs} = \mathcal{Y} + \mathcal{N}$, where $\mathcal{Y}_{obs}$ is the tensor of observations, $\mathcal{Y}$ is the tensor that contains the data to be estimated and $\mathcal{N}$ contains noise with variable variance. The following simulation consists of 400 Monte-Carlo runs with spreading codes of length $I = 4$, a short frame of $J = 30$ QPSK-symbols, $K = 6$ antennas, $L = 2$ interfering symbols, $P = 2$ paths per user and $R = 3$ users, which means that we are on the uniqueness bound defined in (10).

The curves in Figure 2 show the accuracy of the BFM (blind receiver) in terms of Symbol Error Rate (SER), and that of the MMSE (minimum mean-square error) estimator, which assumes perfect knowledge of the channel (tensors $\mathcal{H}_r$ known) and the antenna array response (matrices $\mathbf{A}_r$ known). We also plot the performance of two semi-blind techniques (either $\mathcal{H}_r$ or $\mathbf{A}_r$ known). It turns out that the performance of the BFM receiver is close to the MMSE (the gap between the 2 curves is only 4 dB). Moreover, the gap between the BFM-curve and the curve for the semi-blind receiver that knows

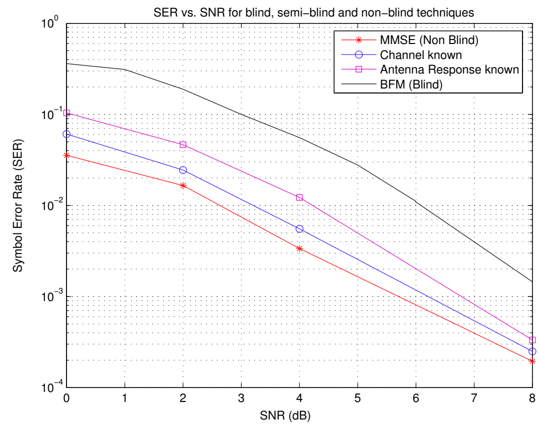


Fig. 2. Performance of the BFM receiver

the antennas response is only 2dB. For scenarios where the uniqueness bound defined in (10) is not reached, we have found an even better precision, with a gap between the BFM and MMSE curve that does not exceed 2dB.

6. CONCLUSION

In this paper, we have shown how Block Factor Analysis of a third-order tensor leads to a powerful blind receiver for multi-user access in wireless communications, with performance close to the MMSE receiver. The tensor model takes both ISI and multi-path propagation aspects into account, which was not the case for the blind PARAFAC receiver in [2]. Our method can also be applied to other systems where three diversities are available (e.g. temporal oversampling instead of code diversity).

7. REFERENCES

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