A TENSOR-BASED BLIND DS-CDMA RECEIVER USING SIMULTANEOUS MATRIX DIAGONALIZATION

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ABSTRACT

In this paper, we consider the problem of blind separation-equalization of DS-CDMA signals, from convolutive mixtures received by an antenna array. We suppose that multipath reflections occur in the far-field of this array and that Inter-Symbol-Interference is caused by large delay spread. Our receiver is deterministic and relies on a third-order tensor decomposition, called decomposition in rank-(L,L,1) terms, which is a generalization of the well-known Parallel Factor (PARAFAC) decomposition. The technique we propose to calculate this decomposition is based on simultaneous matrix diagonalization, which is more accurate than the standard Alternating Least Squares (ALS) algorithm and also allows to blindly identify more users than previously stated.

1. INTRODUCTION

Blind separation of signals impinging on an antenna array is of paramount importance in many commercial and military applications such as source localization, sensor calibration, and eavesdropping. Moreover, most of the blind problems in the literature are formulated in terms of second order algebra and we refer to [1] and references therein for an overview of the existing approaches. The authors of [2] were the first to propose a multilinear algebraic approach to solve the DS-CDMA multiuser blind separation-equalization problem. By fully exploiting the spatial, temporal and code diversities, they have shown that the samples of the received signal can be stored in a third-order tensor (i.e. a cube) that follows the well-known PARAFAC model [3, 4]. Interestingly, the deterministic blind PARAFAC DS-CDMA receiver does not require knowledge of the channel, CDMA-codes, DOA calibration or statistical independence. However, this model is only valid if the multipath reflectors are in the far field of the receive antenna array and if the delay spread is small (i.e. in the order of a few chips), such that Inter-Symbol-Interference (ISI) can be avoided by adopting a "guard chips" or a "discard prefix" strategy.

In this paper, we focus on the more complex situation with ISI caused by large delay spread (i.e. more than one symbol period). We show that the problem can be solved by a decomposition in rank-(L,L,1) terms of the tensor of observations. This multilinear model [5,6] is a generalization of PARAFAC. Moreover, the technique proposed in [5, 6] to calculate this decomposition is an Alternating Least Squares (ALS) algorithm, which is known to be sensitive to local minima and sometimes needs a large number of iterations to converge. We derive here a Simultaneous Diagonalization (SD) algorithm that outperforms ALS and also allows to extract more users' contributions than previously stated.

The article is organized as follows: in Section 2, we recall the discrete-time instantaneous and convolutive data models for the received signal. In Section 3, we introduce some multilinear algebra prerequisites. In Section 4, we discuss the PARAFAC decomposition of a third-order tensor and the decomposition in rank-(L,L,1) terms. In Section 5, we develop a Simultaneous Diagonalization (SD) algorithm to compute the decomposition in rank-(L,L,1) terms. In Section 6, we illustrate the performance by simulation results.

2. DATA MODEL: ANALYTIC FORM

2.1. Instantaneous model

Let us consider R users transmitting at the same time within the same bandwidth, frames of J symbols spread by DS-CDMA codes of length I, towards an array of K antennas. In a direct-path only propagation scenario, the assumption that the channel is noiseless and memoryless leads to the following instantaneous data model without Inter-Chip-Interference:

$$y_{ijk} = \sum_{r=1}^{R} c_i^{(r)} s_j^{(r)} a_k^{(r)}, \tag{1}$$

where y_{ijk} is the sample of the signal received by the k^{th} antenna at the i^{th} chip-sampling instant within the j^{th} symbol period. The scalar $a_k^{(r)}$ is the fading factor between user r and antenna element k, $s_j^{(r)}$ is the j^{th} symbol transmitted by the r^{th} user and $c_i^{(r)}$ is the i^{th} chip of the CDMA code assigned to user r.

2.2. Convolutive model

We now consider a multipath propagation scenario with large delay spread. We assume that for a given user, the multipath channel is the same for all antennas, up to a multiplicative fading factor $a_k^{(r)}$, which is valid when the multipath reflectors are in the far field of the antennas [2, 7]. If we denote by $x_{ijk}^{(r)}$ the i^{th} chip of the signal received by the k^{th} antenna during the j^{th} symbol period for the r^{th} user, we get:

$$x_{ijk}^{(r)} = a_k^{(r)} \sum_{l=1}^{L} h^{(r)}(i + (l-1)I)\, s_{j-l+1}^{(r)}, \tag{2}$$

where $h^{(r)}$ contains the coefficients obtained by convolution between the impulse response of the r^{th} channel and the r^{th} CDMA code. L is the number of interfering symbols. So $h^{(r)}(i + (l-1)I)$ is the coefficient of the overall impulse response at the chip rate corresponding to the i^{th} chip and the l^{th} interfering symbol. We finally get the expression for one sample of the overall received signal by summing the contributions of R users:

$$y_{ijk} = \sum_{r=1}^{R} a_k^{(r)} \sum_{l=1}^{L} h^{(r)}(i + (l-1)I)\, s_{j-l+1}^{(r)}. \tag{3}$$

3. MULTILINEAR ALGEBRA PREREQUISITES

Definition 1. (Tensor) *A multi-way array of which the elements are addressed by N indices is an Nth-order tensor.*

Definition 2. (Mode-n product) *The mode-1 product of a third-order tensor $\mathcal{Y} \in \mathbb{C}^{L\times M\times N}$ by a matrix $\mathbf{A} \in \mathbb{C}^{I\times L}$, denoted by $\mathcal{Y} \bullet_1 \mathbf{A}$, is an ($I \times M \times N$)-tensor with elements defined, for all index values, by*

$$(\mathcal{Y} \bullet_1 \mathbf{A})_{imn} = \sum_{l=1}^{L} y_{lmn} a_{il}$$

Similarly, the mode-2 product of $\mathcal{Y}$ by a matrix $\mathbf{B} \in \mathbb{C}^{J\times M}$ and the mode-3 product by $\mathbf{C} \in \mathbb{C}^{K\times N}$ are the ($L \times J \times N$) and ($L \times M \times K$) tensors, respectively, with elements defined by

$$(\mathcal{Y} \bullet_2 \mathbf{B})_{ljn} = \sum_{m=1}^{M} y_{lmn} b_{jm}$$

$$(\mathcal{Y} \bullet_3 \mathbf{C})_{lmk} = \sum_{n=1}^{N} y_{lmn} c_{kn}$$

In this notation, the matrix product $\mathbf{Y} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^T$ takes the form of $\mathbf{Y} = \mathbf{S} \bullet_1 \mathbf{U} \bullet_2 \mathbf{V}$.

Definition 3. (Rank-1 Tensor) *The third-order tensor $\mathcal{Y} \in \mathbb{C}^{I\times J\times K}$ is rank-1 if its elements can be written as $y_{ijk} = \mathbf{a}_i \mathbf{b}_j \mathbf{c}_k$, where $\mathbf{a} \in \mathbb{C}^{I\times 1}$, $\mathbf{b} \in \mathbb{C}^{J\times 1}$ and $\mathbf{c} \in \mathbb{C}^{K\times 1}$.*

This definition generalizes the definition of a rank-1 matrix: $\mathbf{A} \in \mathbb{C}^{I\times J}$ has rank 1 if $\mathbf{A} = \mathbf{a} \cdot \mathbf{b}^T$.

Definition 4. (Tensor Rank) *The rank of $\mathcal{Y}$ is defined as the minimum number of rank-1 tensors yielding $\mathcal{Y}$ in a linear combination.*

Definition 5. (Kruskal Rank of a Matrix) *The Kruskal rank of a matrix $\mathbf{A}$, denoted by $k(\mathbf{A})$, is defined as the maximal number k such that **any** set of k columns of $\mathbf{A}$ is linearly independent* [8].

4. TENSOR DECOMPOSITIONS

4.1. PARAFAC Decomposition

The PARAllel FACtor (PARAFAC) model or CANonical tensor DECOMPosition (CANDECOMP) was independently introduced in [4] and [9]. It aims at decomposing a tensor as a linear combination of a minimal number R of rank-1 tensors. Let $\mathcal{Y}$ be an $(I \times J \times K)$ tensor, with elements denoted by y_{ijk}. The PARAFAC decomposition of $\mathcal{Y}$ can be written as

$$y_{ijk} = \sum_{r=1}^{R} \mathbf{a}_i^{(r)} \mathbf{b}_j^{(r)} \mathbf{c}_k^{(r)}, \tag{4}$$

where $\mathbf{a}^{(r)}$, $\mathbf{b}^{(r)}$, $\mathbf{c}^{(r)}$ are the r^{th} columns of matrices $\mathbf{A} \in \mathbb{C}^{I\times R}$, $\mathbf{B} \in \mathbb{C}^{J\times R}$ and $\mathbf{C} \in \mathbb{C}^{K\times R}$ respectively, and where i, j and k denote the row index. In [8], Kruskal proved that the PARAFAC decomposition (4) is unique (up to some trivial indeterminacies) if

$$k(\mathbf{A}) + k(\mathbf{B}) + k(\mathbf{C}) \geq 2(R+1). \tag{5}$$

In [2], the authors established the link between this decomposition and the data model of Eq. (1). In fact, this equation can be seen as a PARAFAC decomposition of the tensor of observations $\mathcal{Y} \in \mathbb{C}^{I\times J\times K}$, where each user contribution is characterized by a rank-1 tensor. Eq. (5) should be seen as a bound on R guaranteeing uniqueness of the decomposition. In the next subsection, we show how the convolutive data model of Eq. (3) can algebraically be written as the decomposition in rank-(L,L,1) terms of the third-order tensor of observations.

4.2. Decomposition in rank-(L,L,1) terms

From Eq. (2), for a given user (index r fixed) and for a given antenna (index k fixed), $x_{ijk}^{(r)}$ can be considered as an element of the following $(I \times J)$ matrix $\mathbf{X}_{kr}$

$$\mathbf{X}_{kr} = a_k^{(r)} \left(\mathbf{H}_r \bullet_2 \mathbf{S}_r\right), \tag{6}$$

where $\mathbf{H}_r$ is the $(I \times L)$ matrix with elements defined by $(\mathbf{H}_r)_{i,l} = h^{(r)}(i+(l-1)I)$ and $\mathbf{S}_r$ is the $(J\times L)$ Toeplitz matrix with elements defined by $(\mathbf{S}_r)_{j,l} = s_{j-l+1}^{(r)}$, $i = 1 \ldots I$, $j = 1 \ldots J$ and $l = 1 \ldots L$.

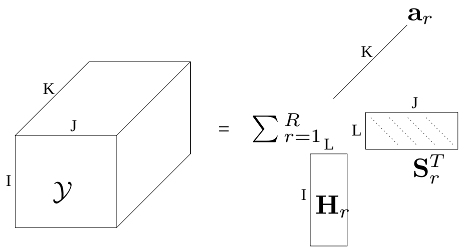

Fig. 1. Schematic representation of the decomposition in rank-(L,L,1) terms

For a given user r, and for all values of indexes i, j, k and l, the sample $x_{ijk}^{(r)}$ can thus be stored in the following third-order tensor $\mathcal{X}_r \in \mathbb{C}^{I\times J\times K}$:

$$\mathcal{X}_r = \mathbf{H}_r \bullet_2 \mathbf{S}_r \bullet_3 \mathbf{a}_r, \tag{7}$$

where $\mathcal{X}_r$ represents the global contribution from a single user, and $\mathbf{a}_r$ is the $(K \times 1)$ vector that contains the fading factors $a_k^{(r)}$ for the K antennas. Finally, we consider R users transmitting at the same time, so we obtain the following tensor equivalent of (3) after summing the R contributions:

$$\mathcal{Y} = \sum_{r=1}^{R} \mathbf{H}_r \bullet_2 \mathbf{S}_r \bullet_3 \mathbf{a}_r. \tag{8}$$

Equation (8) stands for the decomposition of $\mathcal{Y}$ in a sum of rank-(L,L,1) terms [5, 6], and is visualized in Fig. 1. Note that if the delay spread is small (no ISI), i.e., $L = 1$, Eq. (8) is equivalent to PARAFAC.

A generic sufficient condition for uniqueness of this decomposition has been derived in [6]:

$$min\left(\left\lfloor \frac{I}{L} \right\rfloor, R\right) + min\left(\left\lfloor \frac{J}{L} \right\rfloor, R\right) + min\,(K, R) \geq 2R + 2, \tag{9}$$

which implies an upper bound on the number of users that can be allowed at the same time. Let $\mathbf{H}$, $\mathbf{S}$ and $\mathbf{A}$ be the matrices of size $(I \times LR)$, $(J \times LR)$ and $(K \times R)$, respectively resulting from the concatenation of $\mathbf{H}_r$, $\mathbf{S}_r$ and $\mathbf{a}_r$, $r = 1 \ldots R$. From the knowledge of the tensor of observations $\mathcal{Y}$ only, the calculation of the decomposition in rank-(L,L,1) terms consists of the blind estimation of these matrices by minimization of the quadratic cost function

$$\phi(\mathbf{H}, \mathbf{S}, \mathbf{A}) = \|\mathcal{Y} - \sum_{r=1}^{R} \hat{\mathbf{H}}_r \bullet_2 \hat{\mathbf{S}}_r \bullet_3 \hat{\mathbf{a}}_r\|^2.$$

This can be done by means of an Alternating Least Squares (ALS) algorithm [5, 6]. However, this algorithm can be slow and it is sensitive to local minima. Several initializations are thus often needed to find a reliable solution, which increases the computational cost. With respect to this problem, a Simultaneous Diagonalization technique is very attractive.

5. SIMULTANEOUS DIAGONALIZATION

Simultaneous diagonalization (SD) of a set of matrices has become a popular tool in blind signal separation. In [10, 11], the authors have derived a SD algorithm for the PARAFAC decomposition, that outperforms ALS. Moreover, they have shown that this approach implies a new bound on R that is significantly more relaxed than (5). In this section, we generalize the SD technique to the calculation of the decomposition in rank-(L,L,1) terms. We make the following assumptions on the dimensions:

$$\begin{cases} I \geq L \\ J \geq L \\ min(IJ, K) \geq R \end{cases}. \tag{10}$$

Consider an $(I \times J \times K)$ tensor $\mathcal{Y}$ of which the decomposition in rank-(L,L,1) terms is given by

$$\mathcal{Y} = \sum_{r=1}^{R} \mathbf{X}_r \bullet_3 \mathbf{a}_r,$$

in which the $(I \times J)$ matrices $\mathbf{X}_r$ result from

$$\mathbf{X}_r = \mathbf{H}_r \bullet_2 \mathbf{S}_r = \mathbf{H}_r \cdot \mathbf{S}_r^T.$$

Let us build the matrix $\mathbf{Y} \in \mathbb{C}^{JI\times K}$ in which the entries of $\mathcal{Y}$ are stacked as follows:

$$(\mathbf{Y})_{(j-1)I+i,k} = y_{ijk}.$$

This matrix can be seen as the result of row-wise concatenation of the J matrices $(\mathcal{Y})_{:,j,:}$ of size $(I \times K)$. According to the multilinear model under consideration, this matrix can also be written as:

$$\mathbf{Y} = \begin{pmatrix} vec(\mathbf{X}_1) & \cdots & vec(\mathbf{X}_R) \end{pmatrix} \cdot \mathbf{A}^T = \tilde{\mathbf{X}} \cdot \mathbf{A}^T, \tag{11}$$

where the operator vec builds a vector from a matrix by stacking the columns of this matrix one above the other. Under the assumption (10), we can expect the rank of $\mathbf{H}_r \in \mathbb{C}^{I\times L}$, $\mathbf{S}_r \in \mathbb{C}^{J\times L}$ and $\mathbf{X}_r \in \mathbb{C}^{I\times J}$ to be equal to L. Moreover, we can expect the rank of $\tilde{\mathbf{X}} \in \mathbb{C}^{JI\times R}$ and $\mathbf{A} \in \mathbb{C}^{K\times R}$ to be equal to R, which implies that the rank of $\mathbf{Y}$ is R. Consider then the "economy size" SVD of $\mathbf{Y}$**:**

$$\mathbf{Y} = \mathbf{U} \cdot \mathbf{\Sigma} \cdot \mathbf{V}^H, \tag{12}$$

in which $\mathbf{U} \in \mathbb{C}^{JI\times R}$ and $\mathbf{V} \in \mathbb{C}^{K\times R}$ are column-wise orthonormal matrices and in which $\mathbf{\Sigma} \in \mathbb{C}^{R\times R}$ is positive diagonal.

If we put $\mathbf{E} = \mathbf{U} \cdot \mathbf{\Sigma}$, then we deduce from (11) and (12), that there exists an a priori unknown non-singular matrix $\mathbf{W} \in \mathbb{C}^{R\times R}$ that satisfies:

$$\begin{cases} \tilde{\mathbf{X}} &= \mathbf{E} \cdot \mathbf{W} \\ \mathbf{A}^T &= \mathbf{W}^{-1} \cdot \mathbf{V}^H \end{cases}. \tag{13}$$

It is sufficient to estimate the matrix $\mathbf{W}$ to find $\mathbf{H}$, $\mathbf{S}$ and $\mathbf{A}$. Obviously, $\mathbf{A} = \mathbf{V}^* \cdot \mathbf{W}^{-T}$. Furthermore, the columns of $\mathbf{E} \cdot \mathbf{W}$ correspond to the vectorized representation of the $(I \times J)$ matrices $\mathbf{X}_r = \mathbf{H}_r \cdot \mathbf{S}_r^T$ of rank-L. Thus, the columns of $\mathbf{H}_r$ can be estimated as the L left singular vectors associated with the L largest singular values of $\mathbf{X}_r$. The matrix $\mathbf{S}_r$ then corresponds to the product of the first L singular values and the L associated right singular vectors.

The task is now to find $\mathbf{W}$ that satisfies (13). From the matrix $\mathbf{E} \in \mathbb{C}^{JI \times R}$, we build the set of matrices $\mathbf{E}_1, \ldots, \mathbf{E}_R \in \mathbb{C}^{I \times J}$, defined by

$$\mathbf{E}_r = unvec((\mathbf{E})_{:,r}),$$

where $(\mathbf{E})_{:,r}$ is the r^{th} column of $\mathbf{E}$ and $unvec$ is the operator that stacks the entries of a $(JI \times 1)$ vector $\mathbf{u}$ in an $(I \times J)$ matrix $\mathbf{U}$ as follows: $(\mathbf{U})_{i,j} = (\mathbf{u})_{(j-1)I+i}$. We thus have

$$\begin{aligned} \mathbf{E}_r &= unvec((\tilde{\mathbf{X}} \cdot \mathbf{W}^{-1})_{:,r}) \\ &= \sum_{k=1}^{R} (\mathbf{H}_r \cdot \mathbf{S}_r^T)(\mathbf{W}^{-1})_{kr}. \end{aligned} \qquad (14)$$

This means that the matrices $\mathbf{E}_r$ consist of linear combinations of the rank-L matrices $\mathbf{X}_r = \mathbf{H}_r \cdot \mathbf{S}_r^T$. Turned the other way around, we now have to find the linear combinations of the matrices $\mathbf{E}_r$ that yield rank-L matrices, because the coefficients of these linear combinations will yield the matrix $\mathbf{W}$ we are looking for. To solve this problem, we need a tool that allows us to determine whether a matrix is rank-L or not.

For the sake of clarity and to avoid the use of too many indexes, we assume that $L = 2$ in the following. The results can easily be generalized to any value of L. The following generalizes in a non-trivial way the results of [10].

Theorem 1 *Consider the mapping* $\Phi : (\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}) \in \mathbb{C}^{I \times J} \times \mathbb{C}^{I \times J} \times \mathbb{C}^{I \times J} \to \Phi(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}) \in \mathbb{C}^{I \times I \times I \times J \times J \times J}$ *defined by*

$$\begin{aligned} &(\Phi(\mathbf{X}, \mathbf{Y}, \mathbf{Z}))_{ijklmn} \\ &= x_{il} D_{m,n}(y_j, z_k) - x_{i,m} D_{l,n}(y_j, z_k) + x_{i,n} D_{l,m}(y_j, z_k) \\ &+ x_{il} D_{m,n}(z_j, y_k) - x_{i,m} D_{l,n}(z_j, y_k) + x_{i,n} D_{l,m}(z_j, y_k) \\ &+ y_{il} D_{m,n}(x_j, z_k) - y_{i,m} D_{l,n}(x_j, z_k) + y_{i,n} D_{l,m}(x_j, z_k) \\ &+ y_{il} D_{m,n}(z_j, x_k) - y_{i,m} D_{l,n}(z_j, x_k) + y_{i,n} D_{l,m}(z_j, x_k) \\ &+ z_{il} D_{m,n}(x_j y_k) - z_{i,m} D_{l,n}(x_j, y_k) + z_{i,n} D_{l,m}(x_j, y_k) \\ &+ z_{il} D_{m,n}(y_j x_k) - z_{i,m} D_{l,n}(y_j, x_k) + z_{i,n} D_{l,m}(y_j, x_k) \end{aligned} ,$$

where $D_{m,n}(y_j, z_k) = y_{jm} z_{kn} - y_{jn} z_{km}$*. Then we have* $\Phi(\mathbf{X}, \mathbf{X}, \mathbf{X}) = 0$ *if and only if* $\mathbf{X}$ *is at most rank-2.*

Proof: The entries of $\Phi(\mathbf{X}, \mathbf{X}, \mathbf{X})/(3!)$ correspond to the determinants of the different (3×3) submatrices of $\mathbf{X}$. A necessary and sufficient condition for $\mathbf{X}$ to be at most rank-2, is that all these determinants vanish. ■

Let us introduce $\mathcal{P}_{rst} = \Phi(\mathbf{E}_r, \mathbf{E}_s, \mathbf{E}_t)$. Since Φ is trilinear, we have from (14) :

$$\mathcal{P}_{rst} = \sum_{u,v,w=1}^{R} (\mathbf{W}^{-1})_{ur} (\mathbf{W}^{-1})_{vs} (\mathbf{W}^{-1})_{wt} \Phi(\mathbf{E}_u,\ \mathbf{E}_v,\ \mathbf{E}_w). \qquad (15)$$

Assume at this point that there exists a symmetric third-order tensor $\mathcal{M}$ of which the entries m_{rst} satisfy the following set of homogeneous linear equations (we will justify this assumption below):

$$\sum_{r,s,t=1}^{R} m_{rst} \mathcal{P}_{rst} = 0. \qquad (16)$$

We define the set $\mathrm{P} = \{\Phi(\mathbf{E}_u,\ \mathbf{E}_v,\ \mathbf{E}_v) \,|\, 1 \leqslant u \neq v \leqslant R\} \cup \{\Phi(\mathbf{E}_u,\ \mathbf{E}_v,\ \mathbf{E}_w) \,|1 \leqslant u < v < w \leqslant R\}$. If P is linearly independent, then after substitution of (15) in (16) and using the symmetry of Φ and $\mathcal{M}$, we can show that $\mathbf{W}$ is solution of:

$$\mathcal{M} = \mathcal{D} \bullet_1 \mathbf{W} \bullet_2 \mathbf{W} \bullet_3 \mathbf{W}, \qquad (17)$$

in which $\mathcal{D}$ is diagonal. Actually, we can show that *any* diagonal tensor $\mathcal{D}$ generates a tensor $\mathcal{M}$ that satisfies Eq. (16). Hence, if P is linearly independent, the tensors $\mathcal{M}$ form an R-dimensional subspace of the symmetric $(R \times R \times R)$ tensors. Let $\{\mathcal{M}_r\}$ represent a basis of this subspace, known from the kernel of the set P, cf. (16). We have:

$$\begin{aligned} \mathcal{M}_1 &= \mathcal{D}_1 \bullet_1 \mathbf{W} \bullet_2 \mathbf{W} \bullet_3 \mathbf{W} \\ &\vdots \\ \mathcal{M}_R &= \mathcal{D}_R \bullet_1 \mathbf{W} \bullet_2 \mathbf{W} \bullet_3 \mathbf{W} \end{aligned} \qquad (18)$$

in which $\mathcal{D}_1, \ldots, \mathcal{D}_R$ are diagonal. This yields in terms of the matrix slices of $\mathcal{M}_1, \ldots, \mathcal{M}_R$:

$$\begin{aligned} (\mathcal{M}_1)_{:,:,r} &= \mathbf{W} \cdot \Lambda_{1,r} \cdot \mathbf{W}^T \\ &\vdots \\ (\mathcal{M}_R)_{:,:,r} &= \mathbf{W} \cdot \Lambda_{R,r} \cdot \mathbf{W}^T \quad \forall r \end{aligned} \qquad (19)$$

in which $\Lambda_{1,r}, \ldots, \Lambda_{R,r}$, $1 \leqslant r \leqslant R$, are diagonal. The matrix $\mathbf{W}$ can be obtained from (19) by means of any algorithm for joint-diagonalization by congruence of a set of matrices, such as the extended QZ-iteration proposed in [12]. In the following section we will show by simulation results how the crucial assumption that the set P is linearly independent implies in fact a new bound on the maximum number of users R, that is significantly more relaxed than (9).

Remark 1 *For the rank-L detection, we start from the determinants of the different* $(L+1) \times (L+1)$ *submatrices.*

6. SIMULATION RESULTS

In this section, we illustrate the performance of the blind receiver based on the tensor decomposition in terms of rank-(L,L,1). Fig. 2 shows the results obtained from 1000 Monte-Carlo trials with Additive White Gaussian Noise (AWGN), where $I = 6$, $J = 50$ QPSK symbols, $K = 4$ antennas, $L = 2$ interfering symbols and $R = 4$ users. We evaluate,

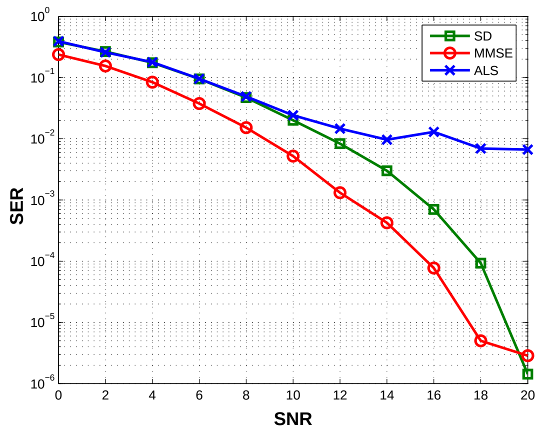

Fig. 2. Performance of the ALS and SD algorithms.

in terms of Symbol Error Rate (SER), the accuracy of the decomposition in rank-(L,L,1) terms, calculated either by ALS with one initialization or by SD. We compare to the performance of the Minimum Mean-Square Error (MMSE) estimator which assumes perfect knowledge of the channel and the antenna array response. It turns out that the performance of the SD-based blind receiver is close to the MMSE receiver (the gap between the two curves is only 2dB at SER=10^{-4}) and outperforms the ALS-based blind receiver. Furthermore, the average time of calculation for SD with these parameters is more than 10 times lower than ALS (both have been compared under the same conditions). Note that we used only one initialization for ALS, so the calculation of the mean SER takes the trials that converged to a local minimum into account. With a sufficient number n of different initializations (typically $n = 5$), the ALS gives approximately the same mean-SER curve as SD but the computation time of ALS is then $10n$ times higher.

The following array gives the maximum number of users' contributions that can be extracted, for different values of the parameters. $R_{max}^{(SC)}$ is the maximum value of R such that the sufficient condition for uniqueness (9) is still satisfied. $R_{max}^{(SD)}$ has been numerically calculated as the maximum value of R such that the set P is linearly independent. These results show that the SD technique implies a new bound on R, that is significantly more relaxed than (9). The mathematical proof for this new bound will be presented in a full paper version of this manuscript.

I	J	K	L	$R_{max}^{(SC)}$	$R_{max}^{(SD)}$
4	4	8	2	2	4
4	5	8	2	2	5
4	6	8	2	3	7

7. CONCLUSION

In this paper, we have proposed to use the third-order tensor decomposition in rank-(L,L,1) terms to build a powerful blind deterministic receiver with performance close to the non-blind MMSE estimator. This receiver can deal with ISI, under the assumption that the multipath reflectors are in the far field. The standard way to compute this multilinear algebraic decomposition is an ALS algorithm, which sometimes converges slowly and is sensitive to local minima. We have shown that it is possible to compute this decomposition by an SD algorithm. This approach is less time-consuming than ALS, more accurate and implies a new bound on the number of contributions that can be extracted.

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