A TENSOR-BASED BLIND DS-CDMA RECEIVER USING SIMULTANEOUS MATRIX DIAGONALIZATION

Dimitri Nion[(1)], Lieven De Lathauwer[(1)],

[(1)]ETIS, UMR 8051 (CNRS,ENSEA,UCP), Cergy-Pontoise, France

E-mail: {nion, delathau}@ensea.fr

We present a deterministic tensor-based technique for the blind separation-equalization of DS-CDMA signals received by an antenna array, in the context of far-field reflections only.
Our method relies on the decomposition in terms of rank-(L,L,1) of a third-order tensor. We show that this decomposition can calculated by means of simultaneous diagonalization of a set of matrices, which is more accurate than the standard ALS algorithm.

Communication System

- Parameters and propagation model:
  - R: Nb of users, transmitting at the same time within the same bandwidth.
  - I: Spreading Factor of CDMA codes.
  - J: Duration of the observation window (in Symbol Periods).
  - K: Nb of receiving Antennas.
  - $I \times J \times K$ samples collected at the receiver
  - Multipath propagation: far-field reflections only (no angular spread) and ISI over L consecutive symbols (large delay spread).

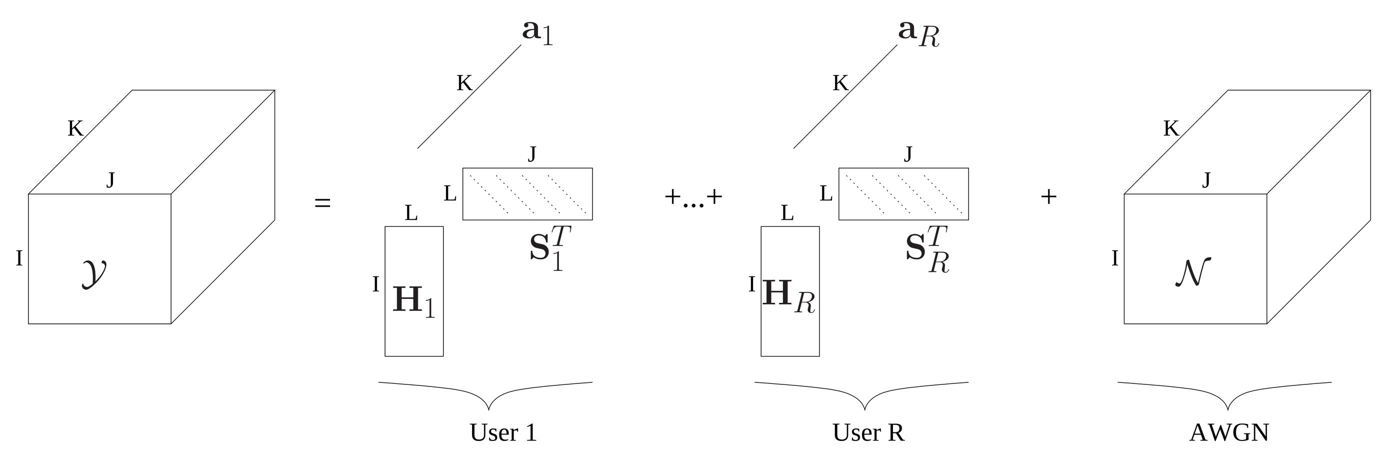
- Chip-Rate Sampled Received Signal: Analytic Form

contribution of R users — ISI over L consecutive symbols — Transmitted symbol (BPSK,QPSK)

$$y_{ijk} = \sum_{r=1}^{R} a_k^{(r)} \sum_{l=1}^{L} h^{(r)}(i+(l-1)I)\, s_{j-l+1}^{(r)} + n_{ijk}$$

received sample: chip i, symbol j, antenna k — response of antenna k — channel impulse response — AWGN sample

- Chip-Rate Sampled Received Signal: Algebraic Form



Decomposition in rank-(L,L,1) terms

- Each term of the decomposition contains the information related to one particular user (channel, antenna response and symbols).
- The matrices $\mathbf{S}_r$ are Toeplitz.

Computation of the Decomposition by Simultaneous Diagonalization

- Optimization problem: From the knowledge of the tensor of observations $\mathcal{Y}$ only, estimate the unknowns $\mathbf{H}_r$, $\mathbf{a}_r$ and $\mathbf{S}_r$ by minimization of the cost function

$$f(\mathbf{H},\mathbf{S},\mathbf{A}) = \|\mathcal{Y} - \hat{\mathcal{Y}}\|_F^2 = \|\mathcal{Y} - \sum_{r=1}^{R} \hat{\mathbf{H}}_r \bullet_2 \hat{\mathbf{S}}_r \bullet_3 \hat{\mathbf{a}}_r\|_F^2 \quad (1)$$

- Assumptions on the dimensions:

$$\begin{cases} I \geq L \\ J \geq L \\ min(IJ,K) \geq R \end{cases}. \quad (2)$$

- Reformulation of the problem:

$$\hat{\mathcal{Y}} = \sum_{r=1}^{R} \hat{\mathbf{X}}_r \bullet_3 \hat{\mathbf{a}}_r,$$

in which the $(I \times J)$ matrices $\hat{\mathbf{X}}_r$ result from $\hat{\mathbf{X}}_r = \hat{\mathbf{H}}_r \bullet_2 \hat{\mathbf{S}}_r = \hat{\mathbf{H}}_r \cdot \hat{\mathbf{S}}_r^T$.

  - Consider one matrix representation of $\hat{\mathcal{Y}}$: $\hat{\mathbf{Y}} \in \mathbb{C}^{JI \times K}$

$$\hat{\mathbf{Y}} = \left(vec(\hat{\mathbf{X}}_1) \cdots vec(\hat{\mathbf{X}}_R) \right) \cdot \hat{\mathbf{A}}^T = \tilde{\mathbf{X}} \cdot \hat{\mathbf{A}}^T \quad (3)$$

  - SVD of $\hat{\mathbf{Y}}$:

$$\mathbf{Y} = \mathbf{U} \cdot \mathbf{\Sigma} \cdot \mathbf{V}^H = \mathbf{E} \cdot \mathbf{V}^H \quad (4)$$

  - Combine Eqs. (3) and (4): there exists an a priori unknown non-singular matrix $\mathbf{W} \in \mathbb{C}^{R \times R}$ that satisfies

$$\begin{cases} \tilde{\mathbf{X}} &= \mathbf{E} \cdot \mathbf{W} \\ \hat{\mathbf{A}}^T &= \mathbf{W}^{-1} \cdot \mathbf{V}^H \end{cases}. \quad (5)$$

- How to find $\mathbf{W} \in \mathbb{C}^{R \times R}$?
  - The matrix $\mathbf{E}$ is $(JI \times R)$. We denote by $\mathbf{E}_r$ the $(I \times J)$ matrix representation of the r^{th} column of $\mathbf{E}$. We have:

$$\underbrace{\left(vec(\hat{\mathbf{X}}_1) \cdots vec(\hat{\mathbf{X}}_R) \right)}_{JI \times R} = \underbrace{\left(vec(\mathbf{E}_1) \cdots vec(\mathbf{E}_R) \right)}_{JI \times R} \underbrace{\mathbf{W}}_{R \times R}$$

For $r = 1 \dots R$, we thus have

$$\underbrace{\hat{\mathbf{X}}_r}_{\text{Rank L matrix}} = \mathbf{W}_{1r}\, \mathbf{E}_1 + \dots + \mathbf{W}_{Rr}\, \mathbf{E}_R$$

  - The coefficients of $\mathbf{W}$ are those of linear combinations of the matrices $\mathbf{E}_r$ that yield the rank-L matrices $\hat{\mathbf{X}}_r$.
  - Tool: mapping for rank-L detection (cf paper). Let $\hat{\mathbf{X}} \in \mathbb{C}^{I \times J}$,

$$\phi\,\underbrace{(\hat{\mathbf{X}}, \hat{\mathbf{X}}, \dots, \hat{\mathbf{X}})}_{(L+1)\,times} = 0 \; if\, and\, only\, if\; \hat{\mathbf{X}}\, is\, at\, most\, rank - L$$

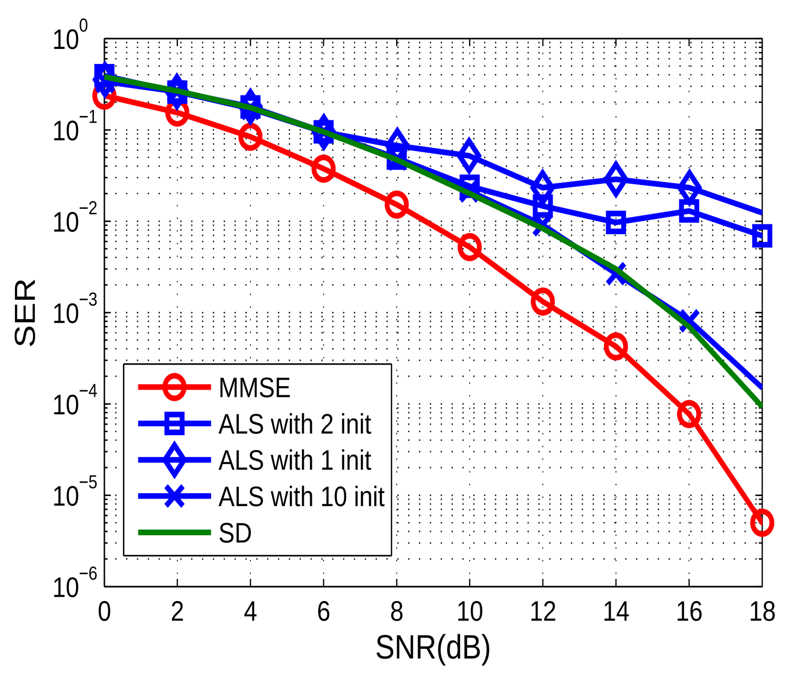
  - After some algebraic manipulations (see paper for details), we can show that the matrix $\mathbf{W}$ can be estimated by simultaneous diagonalization of the following set of matrices:

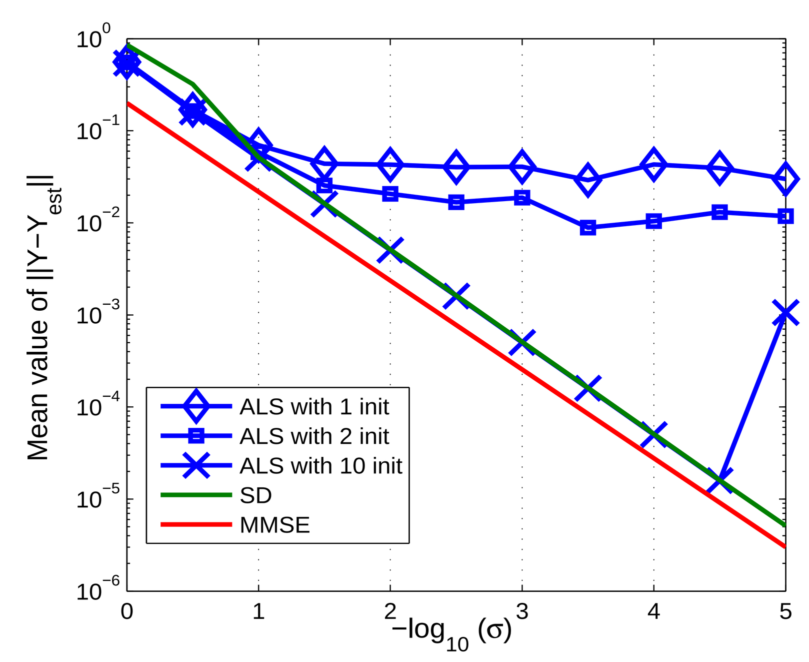
$$\underbrace{\mathbf{M}_1}_{\text{known matrices}} = \mathbf{W} \cdot \underbrace{\mathbf{D}_1}_{\text{diagonal matrices}} \cdot \mathbf{W}^T, \quad \dots, \quad \mathbf{M}_R = \mathbf{W} \cdot \mathbf{D}_R \cdot \mathbf{W}^T$$

  - The system can be solved by any algorithm for joint-diagonalization by congruence of a set of matrices.
  - Once $\mathbf{W}$ is found, the estimation of $\mathbf{A}$ is given by $\hat{\mathbf{A}} = \mathbf{V}^* \cdot \mathbf{W}^{-T}$. The columns of $\hat{\mathbf{H}}_r$ can be estimated as the L left singular vectors associated with the L largest singular values of $\hat{\mathbf{X}}_r$. The matrix $\hat{\mathbf{S}}_r$ then corresponds to the product of the first L singular values and the L associated right singular vectors.

Simulation Results

- Parameters: codes of length $I = 8$, $J = 50$ QPSK symbols collected, $K = 4$ antennas, $L = 2$ interfering symbols, $R = 4$ users.





- The SD technique implies a new bound for the number of users (R_{max}^{SD}), more relaxed than the sufficient condition previously derived (R_{max}^{SC}).

I	J	K	L	$R_{max}^{(SC)}$	$R_{max}^{(SD)}$
4	4	8	2	2	4
4	5	8	2	2	5
4	6	8	2	3	7

SPAWC 2007 June 17-20 2007 Helsinki

IEEE Signal Processing Society