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A time-frequency technique for blind separation and localization of pure delayed sources

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We present a time-frequency technique for the blind separation and localization of several sources, where a single scaled and delayed version of each source contributes to each sensor recording. The separation is performed in the time-frequency domain via an Alternating Least Squares (ALS) algorithm coupled with a Vandermonde structure enforcing strategy. The Time Differences Of Arrival (TDOAs) estimates are then exploited to localize the sources individually.

Problem Formulation

- Parameters and propagation model:
  - N: nb. of sources $s_n(t)$, $n = 1, \dots, N$.
  - $M \geq N$: nb. of sensors; received signals $r_m(t)$, $m = 1, \dots, M$.
  - a_{mn}: attenuation factor between nth source and mth sensor.
  - τ_{mn}: Time Of Arrival (TOA), in seconds, between nth source and mth sensor.
  - Linear time-shift mixing model: line of sight propagation, no reflections [1].
- Time-shift model:

$$r_m(t) = \sum_{n=1}^{N} a_{mn} s_n(t - \tau_{mn}) \tag{1}$$

Time-frequency reformulation

- Parameters:
  - F: Length of each DFT frame, $f = 1, \dots, F$.
  - P: nb. of (possibly overlapping) frames, $p = 1, \dots, P$.
  - $r_m(p, f)$: (p, f)th time-frequency sample of the mth recording.
  - $s_n(p, f)$: (p, f)th time-frequency sample of the nth source.
  - D_{mn}: Time Of Arrival (TOA), in number of samples, between nth source and mth sensor.
  - $\omega = \exp(-2j\pi/F)$.
- Time-Frequency discrete mixing model:
  - Analytic model:

$$r_m(p, f) \simeq \sum_{n=1}^{N} a_{mn} \omega^{(f-1)D_{mn}} s_n(p, f), \quad f = 1, \dots, F. \tag{2}$$

  - Matrix format:

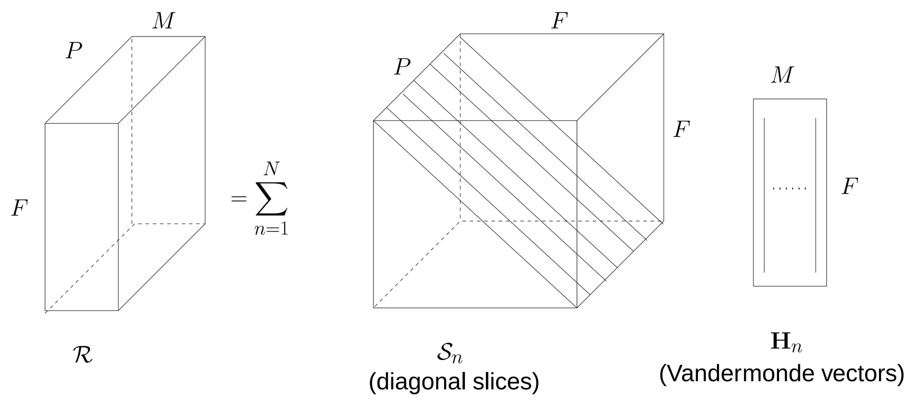
$$\mathbf{R}(f) \simeq \mathbf{H}(f) \cdot \mathbf{S}(f), \quad f = 1, \dots, F, \tag{3}$$

  where
  - $\star$ $[\mathbf{R}(f)]_{m,p} \stackrel{\text{def}}{=} r_m(p, f)$ is the $M \times P$ time-frequency observed matrix,
  - $\star$ $[\mathbf{S}(f)]_{n,p} \stackrel{\text{def}}{=} s_n(p, f)$ is the $N \times P$ rank-N time-frequency source matrix,
  - $\star$ $[\mathbf{H}(f)]_{m,n} \stackrel{\text{def}}{=} a_{mn} \omega^{(f-1)D_{mn}}$ is the $M \times N$ rank-N mixing matrix.
  - Additional structure: Vandermonde vectors

$$\mathbf{h}_{mn} = [\mathbf{H}_{mn}(1), \mathbf{H}_{mn}(2), \dots, \mathbf{H}_{mn}(F)]^T = \left[a_{mn}, a_{mn}\omega^{D_{mn}}, \dots a_{mn}\omega^{(F-1)D_{mn}}\right]^T \tag{4}$$

  - Tensor format:

$$\mathcal{R} = \sum_{n=1}^{N} \mathcal{S}_n \bullet_2 \mathbf{H}_n^T. \tag{5}$$



- Model ambiguities:

$$\mathcal{R} = \sum_{n=1}^{N} (\mathcal{S}_n \bullet_2 \mathbf{Z}_n^{-1}) \bullet_2 (\mathbf{H}_n^T \cdot \mathbf{Z}_n). \tag{6}$$

  - To preserve the whole structure, $\mathbf{Z}_n$ has to be diagonal and $\mathbf{u}_n \stackrel{\text{def}}{=} \text{diag}(\mathbf{Z}_n)$ has to be a Vandermonde vector:

$$\tilde{\mathbf{H}}_n = \text{diag}([\alpha_n, \alpha_n\omega^{\phi_n}, \dots, \alpha_n\omega^{(F-1)\phi_n}])\mathbf{H}_n, \tag{7}$$

  with unknown arbitrary scaling factor α_n and phase factor ϕ_n.
  - In case of perfect separation, we get the estimates: $\tilde{a}_{mn} \stackrel{\text{def}}{=} a_{mn}\alpha_n$ and $\tilde{D}_{mn} \stackrel{\text{def}}{=} D_{mn} + \phi_n$.
  - The ambiguities $\{\alpha_n, \phi_n\}$ only depend on the source and can be removed by choosing a reference sensor, say $\tilde{M}$, and work with the relative attenuation factor $a_{mn}^{(rel)} \stackrel{\text{def}}{=} \frac{\tilde{a}_{mn}}{\tilde{a}_{\tilde{M}n}} = \frac{a_{mn}}{a_{\tilde{M}n}}$ and the relative Time Difference Of Arrival (TDOA) $D_{mn}^{(rel)} \stackrel{\text{def}}{=} \tilde{D}_{mn} - \tilde{D}_{\tilde{M}n} = D_{mn} - D_{\tilde{M}n}$.

ALS algorithm with Vandermonde structure

- Cost function and optimization problem

$$\gamma \stackrel{\text{def}}{=} \sum_{f=1}^{F} \|\mathbf{R}(f) - \mathbf{H}(f) \cdot \mathbf{S}(f)\|^2. \tag{8}$$

$$\min_{\{\mathbf{H}(f), \mathbf{S}(f)\}_{f=1}^{F}} \gamma$$

s.t. $\mathbf{h}_{mn}$ defined in (4) is a Vandermonde vector, $\forall m, \forall n$,

STEP 1: Time-frequency computation

Build $\mathbf{R}(f) \in \mathbb{C}^{M \times P}$, $f = 1, \dots, F$ from FFT of P overlapping windowed frames of recorded signals. (Typical parameters: $F = 2048$, Hanning window, 50% overlap).

STEP 2: Blind separation

—— Initialization ———-

stop=0, $k = 1$, K_{max} (e.g., $K_{max} = 200$) and ϵ (e.g., $\epsilon = 10^{-6}$). Randomly generate $\hat{\mathbf{S}}(f) \in \mathbb{C}^{N \times P}$, $f = 1, \dots, F$. Possibly try several random starting points.

—– Start alternating updates ———

while stop=0

$k = k + 1$

(2.a). $\hat{\mathbf{H}}^{(LS)}(f) = \mathbf{R}(f) \cdot \hat{\mathbf{S}}(f)^\dagger, \quad f = 1, \dots, F.$

(2.b). $\{\hat{\tilde{D}}_{mn}, \hat{\tilde{a}}_{mn}\} \leftarrow$ periodogram$(\hat{\mathbf{h}}_{mn}^{(LS)}), \quad m = 1, \dots, M, \ n = 1, \dots, N.$, see [2].

$\hat{\mathbf{h}}_{mn}^{(VDM)} \leftarrow [\hat{\tilde{a}}_{mn}, \hat{\tilde{a}}_{mn}w^{\hat{\tilde{D}}_{mn}}, \dots, \hat{\tilde{a}}_{mn}w^{(F-1)\hat{\tilde{D}}_{mn}}], \quad m = 1, \dots, M, \ n = 1, \dots, N.$

(2.c). $\hat{\mathbf{S}}(f) = \hat{\mathbf{H}}^{(VDM)}(f)^\dagger \cdot \mathbf{R}(f), \quad f = 1, \dots, F.$

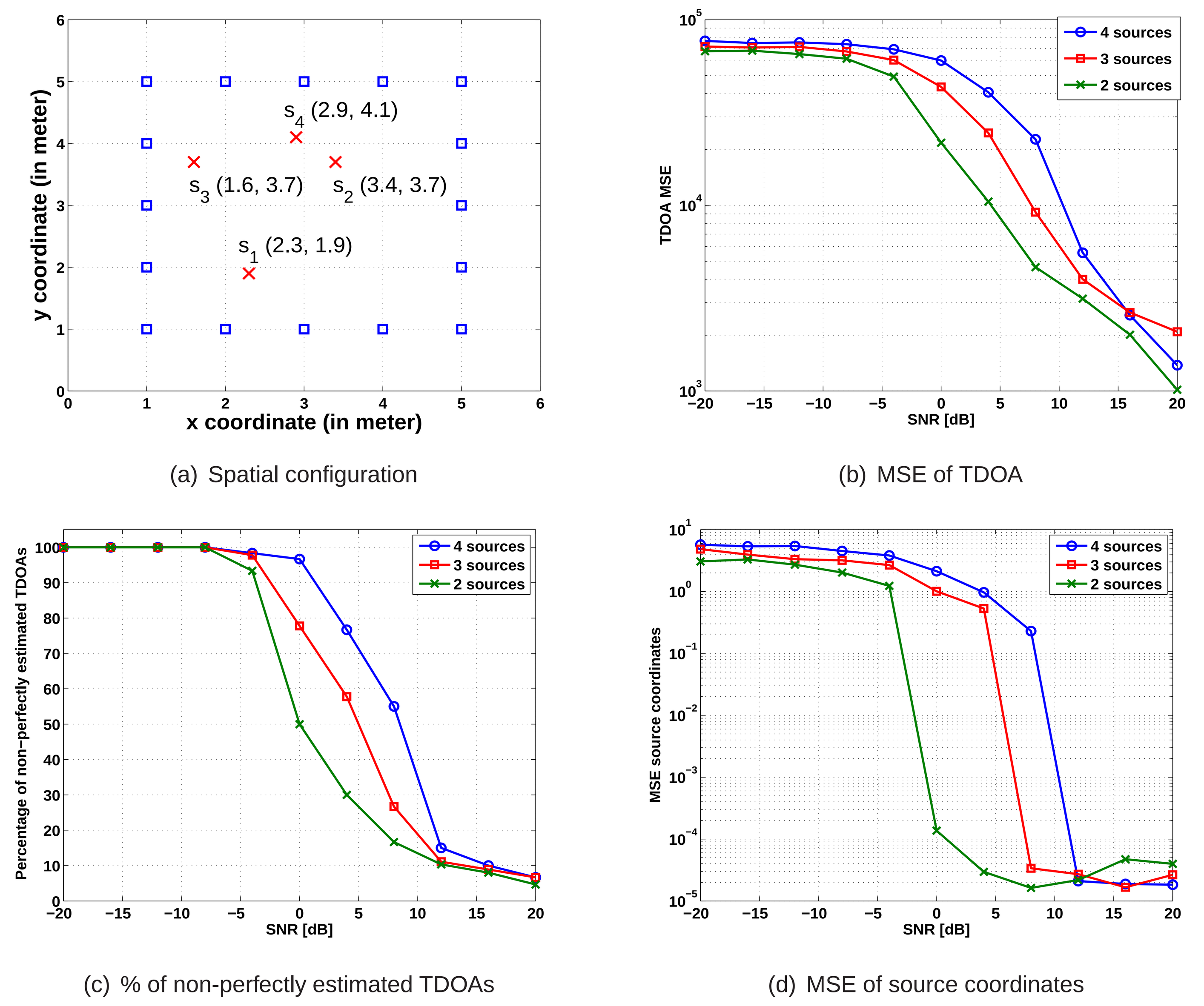
if ($k = K_{max}$) or ($|\gamma^{(k)} - \gamma^{(k-1)}| \leq \epsilon$);stop=1;end

end

STEP 3: Blind localization

- Choose ref. sensor $\tilde{M}$ and compute TDOAs $\hat{D}_{mn}^{(rel)} = \hat{\tilde{D}}_{mn} - \hat{\tilde{D}}_{\tilde{M}n}$.
- Each source is localized individually on the basis of its TDOAs; its x and y coordinates can be estimated in the least squares sense, see [3,4].

Numerical experiments



(a) Spatial configuration (b) MSE of TDOA (c) % of non-perfectly estimated TDOAs (d) MSE of source coordinates

Figure 1: Spatial configuration and results of Monte-Carlo experiments.

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