A PARAFAC-based technique for detection and localization of multiple targets in MIMO radar systems

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Content of the talk

Context: MIMO radar system.

Problem: Detection and localization of multiple targets present in the same range-bin.

State of the art: Radar-imaging localization methods (e.g. Capon, MUSIC)

Limits: Radar-imaging fails for closely spaced targets + sensitivity to Radar Cross Section (RCS) fluctuations

Contribution: Novel method, deterministic, exploits multilinear algebraic structure of received data
→ PARAFAC Decomposition of an observed tensor

Roadmap

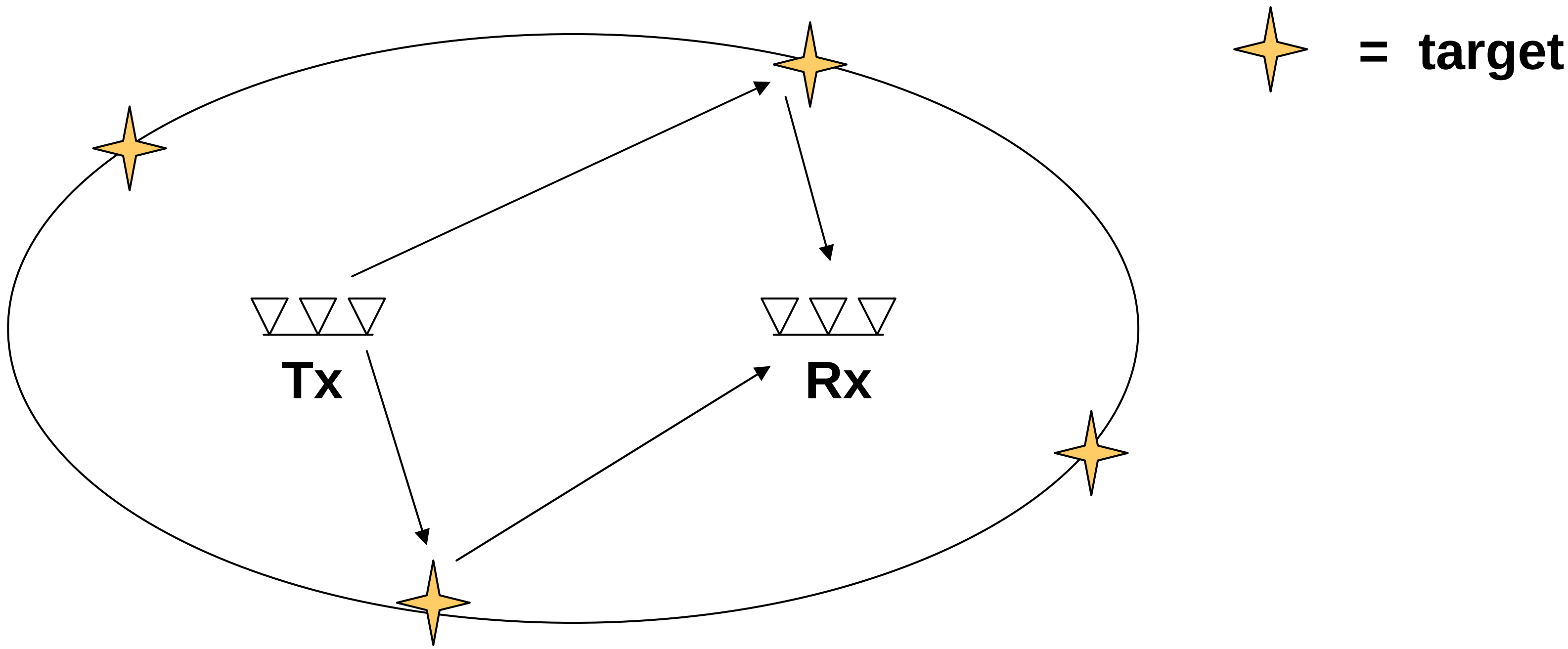
I. Introduction (problem statement + data model)

II. State of the art (Localization via Capon beamforming and MUSIC)

III. Localization via PARAFAC

IV. Conclusion and perspectives

I. Introduction: problem statement



→ K targets in the same range-bin

→ Transmitter Tx and receiver Rx equipped with closely spaced antennas

→ Target = a point source in the far field

<u>Problem :</u> estimate the number of targets and their DODs and DOAS

I. Introduction: parameters

M_t transmit colocated antennas
M_r receive colocated antennas
K targets in the range-bin of interest
$\mathbf{A}(\theta)=[\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K)]$ the M_txK transmit steering matrix
$\mathbf{B}(\alpha)=[\mathbf{b}(\alpha_1), \ldots, \mathbf{b}(\alpha_K)]$ the M_rxK receive steering matrix
$\mathbf{S}=[s_1(t); s_2(t); \ldots; s_{Mt}(t)]$ is M_txL, holds M_t mutually orthogonal transmitted pulse waveforms, with L samples per pulse period
Q consecutive pulses are transmitted
β_{kq} RCS reflection coeff. of target k during pulse q

I. Introduction: data model

Assumption : Swerling case II target model

« Receive and Transmit steering matrices **B**(α) and **A**(θ) constant over the duration of Q pulses while the target reflection coefficients β_{kq} are varying independently from pulse to pulse».

$$\mathbf{X}_q = \mathbf{B}(\alpha)\underbrace{diag([\beta_{1q},\ldots,\beta_{Kq}])}_{=\Sigma_q}\mathbf{A}^T(\theta)\mathbf{S} + \mathbf{W}_q, \quad q = 1,\ldots,Q$$

M_r x L received data

→Times of arrival known (targets in the same range-bin).

→ Right multiply by (1/L)$\mathbf{S}^H$ and simplify (1/L)$\mathbf{S}\mathbf{S}^H = \mathbf{I}$

$$\mathbf{Y}_q = \mathbf{B}(\alpha)\Sigma_q\mathbf{A}^T(\theta) + \mathbf{Z}_q, \quad q = 1,\ldots,Q$$

M_r x M_t received data
after matched filtering

II. State of the art: single-pulse radar-imaging

Radar-imaging techniques working on per-pulse basis:

$$\mathbf{X}_q = \mathbf{B}(\alpha)\Sigma_q \mathbf{A}^T(\theta)\mathbf{S} + \mathbf{W}_q, \quad q = 1,\ldots,Q$$

→ Beamforming techniques [Xu, Li & Stoica].

Example: Capon Beamforming . Suppose colocated arrays (α=θ).

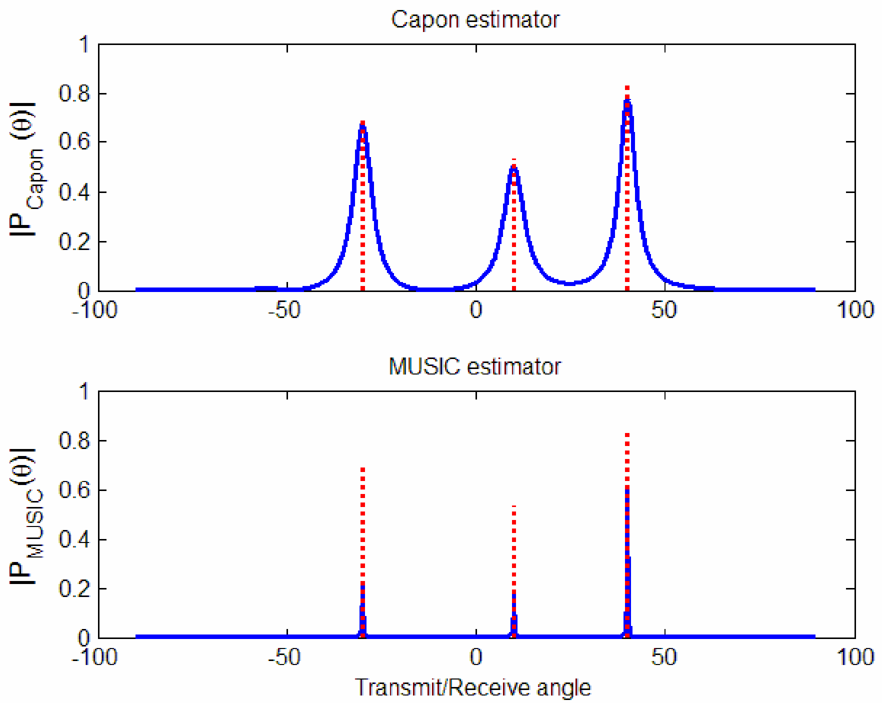
$$\hat{\beta}(\theta) = \frac{\mathbf{b}^H(\theta)\mathbf{R}_{XX}^{-1}\mathbf{X}_q\mathbf{S}^H\mathbf{a}^*(\theta)}{L[\mathbf{b}^H(\theta)\mathbf{R}_{XX}^{-1}\mathbf{b}(\theta)][\mathbf{a}^T(\theta)\mathbf{a}^*(\theta)]}, \qquad \mathbf{R}_{XX} = \frac{1}{L}\mathbf{X}_q\mathbf{X}_q^H$$

→ MUSIC estimator.

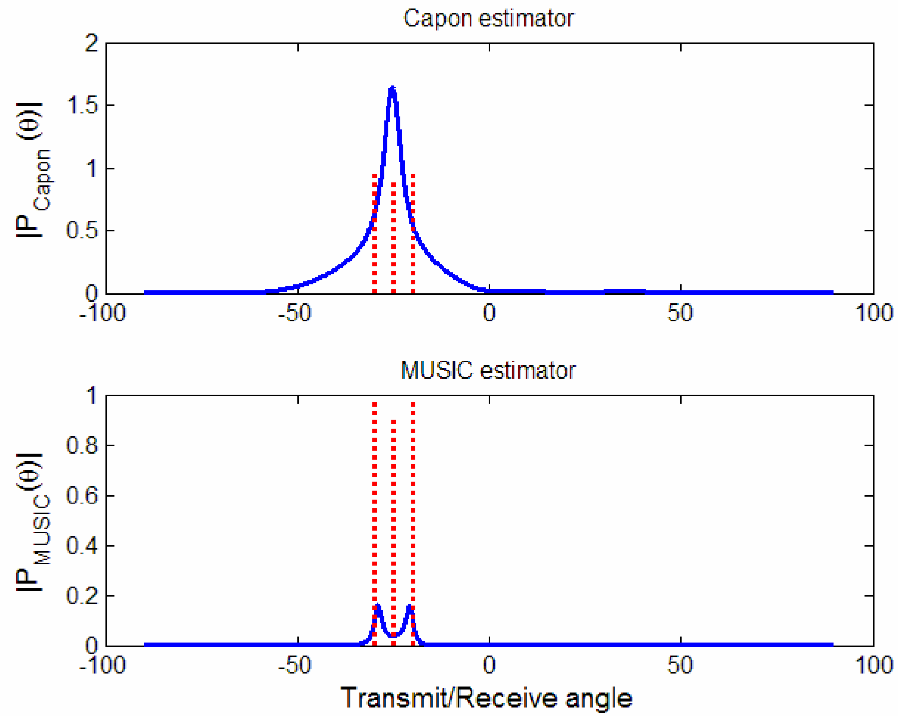
$$P_{MUSIC}(\theta) = \frac{1}{\mathbf{b}^H(\theta)\mathbf{E}_w\mathbf{E}_w^H\mathbf{b}(\theta)}, \quad \mathbf{E}_w = \text{noise eigenvectors of } \mathbf{R}_{\mathbf{XX}}$$

II. State of the art: single-pulse radar-imaging

Typical Capon and MUSIC spectra for a given pulse



Widely spaced targets (-30°,10°,40°)



Closely spaced targets (-30°,-25°,-20°)

- ❑ <u>Problem 1:</u> single lobe occurs for closely located targets
- ❑ <u>Problem 2:</u> update spectrum for each new pulse → scintillation due to fading (fluctuations of RCS coeff. from pulse to pulse)

II. State of the art: multiple-pulses radar-imaging

Q : Mitigate RCS fluctuations? → first need a multi-pulse data model

$$\mathbf{Y}_q = \mathbf{B}(\alpha) \; \Sigma_q \; \mathbf{A}^T(\theta) + \mathbf{Z}_q$$

vectorize

$$\mathbf{y}_q = [\mathbf{a}(\theta_1) \otimes \mathbf{b}(\alpha_1), \ldots, \mathbf{a}(\theta_K) \otimes \mathbf{b}(\alpha_K)][\beta_{q1}, \ldots, \beta_{qK}]^T + \mathbf{z}_q$$

$$= \mathbf{A}(\theta) \odot \mathbf{B}(\alpha) \qquad = \mathbf{c}_q^T$$

Q pulses (concatenation)

$$\mathbf{Y} = [\mathbf{A}(\theta) \odot \mathbf{B}(\alpha)] \, \mathbf{C}^T + \mathbf{Z}$$

II. State of the art: multiple-pulses radar-imaging

Radar-imaging techniques working on a multi-pulse basis:

$$\mathbf{Y} = [\mathbf{A}(\theta) \odot \mathbf{B}(\alpha)]\, \mathbf{C}^T$$

- Capon beamforming *[Yan, Li, Liao]*

$$P_{Capon}(\theta, \alpha) = \frac{1}{(\mathbf{a}(\theta) \otimes \mathbf{b}(\alpha))^H \mathbf{R}_{YY}^{-1} (\mathbf{a}(\theta) \otimes \mathbf{b}(\alpha))}$$

- MUSIC

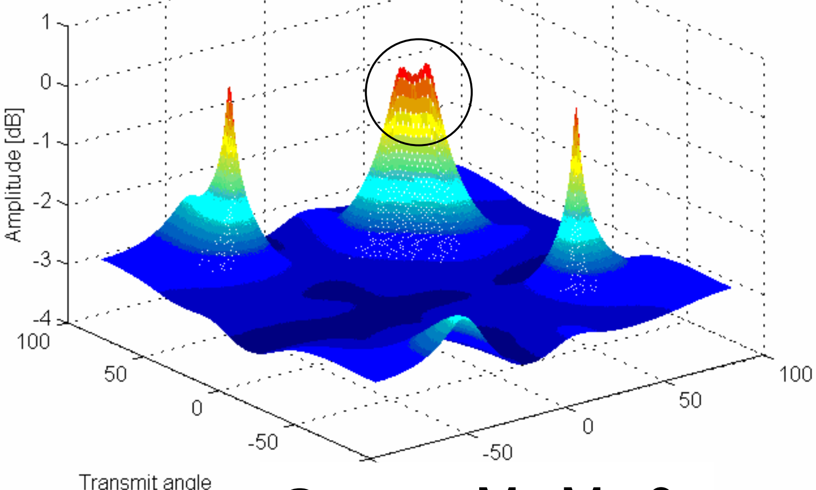
$$P_{MUSIC}(\theta, \alpha) = \frac{1}{(\mathbf{a}(\theta) \otimes \mathbf{b}(\alpha))^H \mathbf{E}_w \mathbf{E}_w^H (\mathbf{a}(\theta) \otimes \mathbf{b}(\alpha))}$$

$\mathbf{E}_w =$ noise eigenvecto rs of $\mathbf{R}_{YY}$

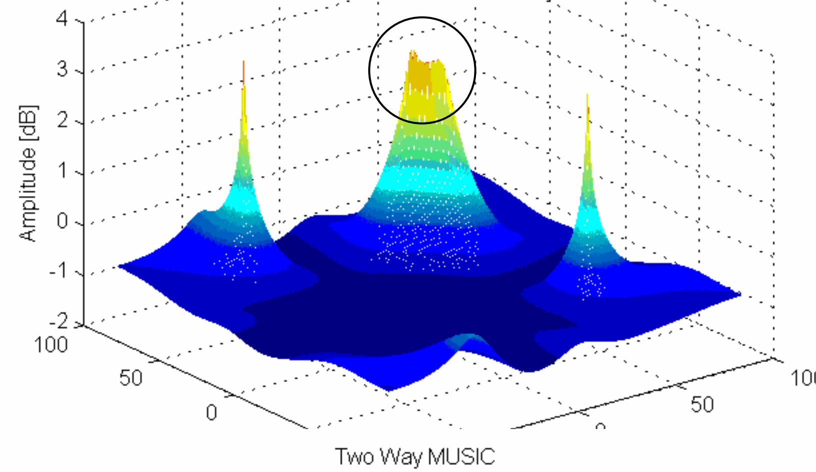
II. State of the art: multiple-pulses radar-imaging

$$K = 5, \{\theta_k\} = \{40°, 35°, 30°, -40°, 65°\}, \{\alpha_k\} = \{20°, 25°, 30°, 50°, -45°\}$$

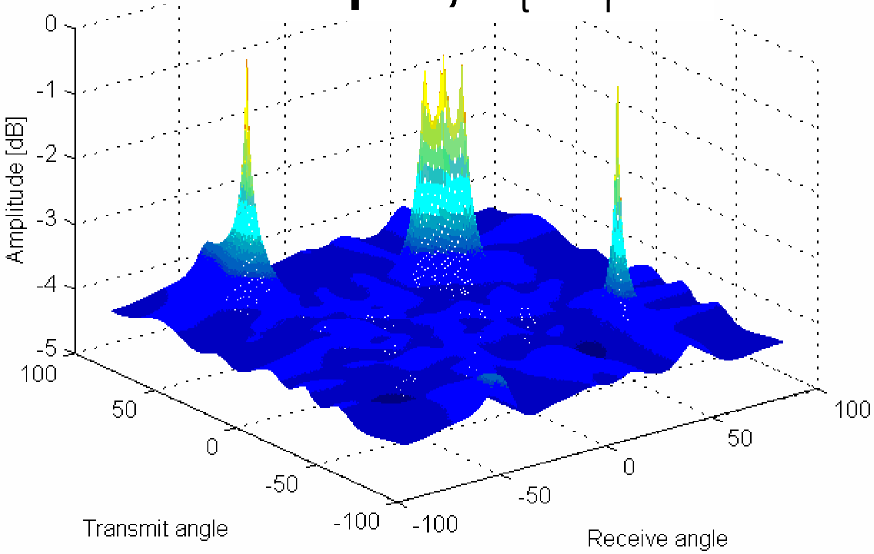
Capon, $M_t = M_r = 4$



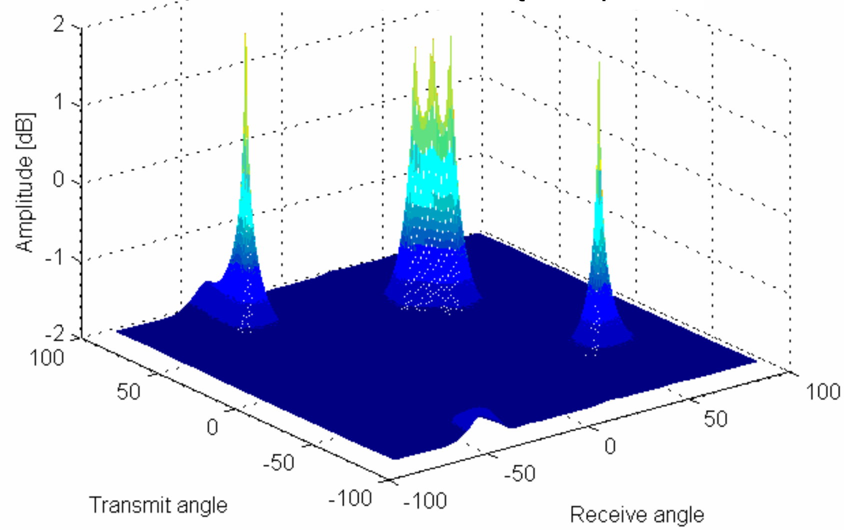
MUSIC, $M_t = M_r = 4$



Capon, $M_t = M_r = 9$



MUSIC, $M_t = M_r = 9$



III. Localization via PARAFAC: overview

Problems:

- Capon and MUSIC 2D-imaging work on multi-pulse basis **but** fail if no distinguishable lobe for each target (e.g. closely located targets)

- Capon and MUSIC spectra have to be computed for each pair of angles
  → time consuming for dense angular grid

→ **Our contribution:** starting from the same data model,

$$\mathbf{Y} = [\mathbf{A}(\theta) \odot \mathbf{B}(\alpha)]\, \mathbf{C}^T$$

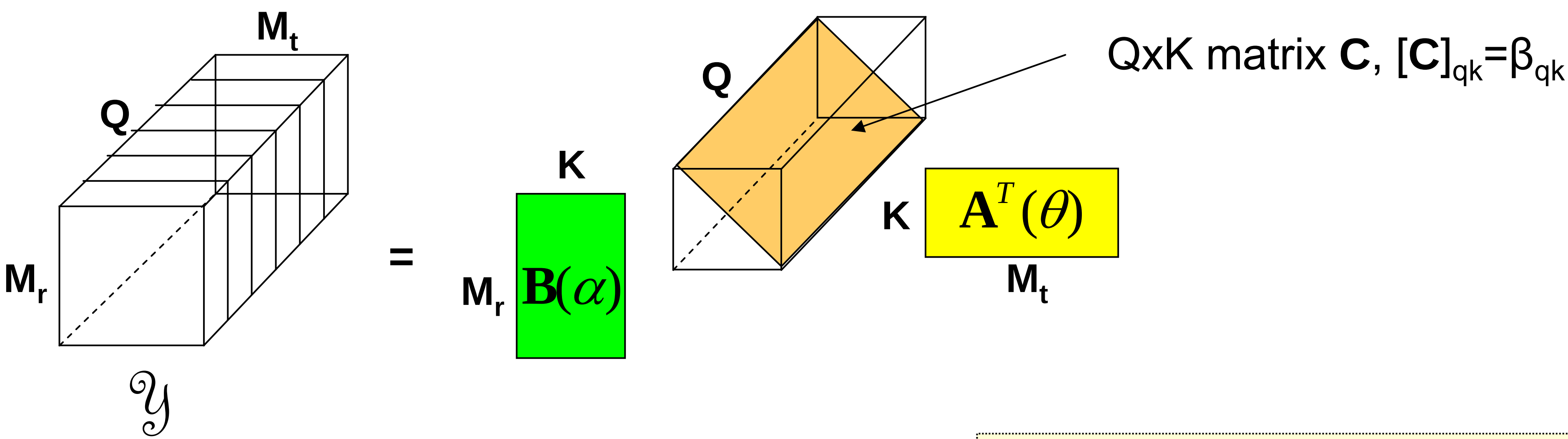
exploitation of the algebraic structure of **Y** is sufficient for blind estimation of **A**(θ), **B**(α) and **C.**

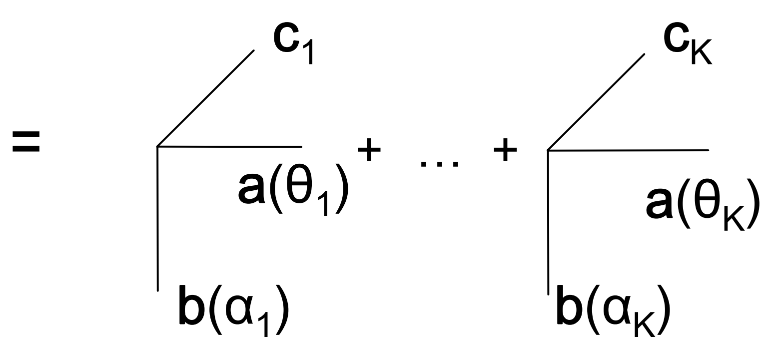
Indeed Y follows the well-known PARAFAC model.

III. Localization via PARAFAC: model

$$\mathbf{Y}_q = \mathbf{B}(\alpha)\,\boldsymbol{\Sigma}_q\,\mathbf{A}^T(\theta) + \mathbf{Z}_q$$

M_r x M_t matrix observed Q times, q=1,…,Q. **B**(α) and **A**(θ) fixed over Q pulses.





PARAFAC decomposition:
$\mathcal{Y}$ =Sum of K rank-1 tensors.

Each target contribution is a rank-1 tensor

III. Localization via PARAFAC: summary

❑ Given the (M_rxM_txQ) tensor $\mathcal{Y}$, compute its PARAFAC decomposition in K terms to estimate **A**(θ), **B**(α) and **C**.

→ Several algorithms in the literature (e.g. Alternating Least Squares (ALS), ALS+Enhanced Line Search, Levenberg-Marquardt, Simultaneous Diagonalization, …)

❑ **Key point:** under some conditions (next slide), PARAFAC is unique up to trivial indeterminacies:

➢ Columns of **A**(θ), **B**(α) and **C** arbitrarily permuted (same permutation)

➢ Columns of **A**(θ), **B**(α) and **C** arbitrarily scaled (scaling factor removed by recovering the known array manifold structure on the steering matrices estimates, after which the DODs and DOAs are extracted).

III. Localization via PARAFAC: uniqueness

- **Condition 1: A**(θ) and **B**(α) full rank and **C** full-column rank. If

$$K \geq 2 \text{ and } M_t(M_t - 1)M_r(M_r - 1) \geq 2K(K-1)$$

then uniqueness is guaranteed a.s. *[De Lathauwer].*

- **Condition 2: A**(θ) and **B**(α) are full rank Vandermonde matrices and **C** full-column rank. If

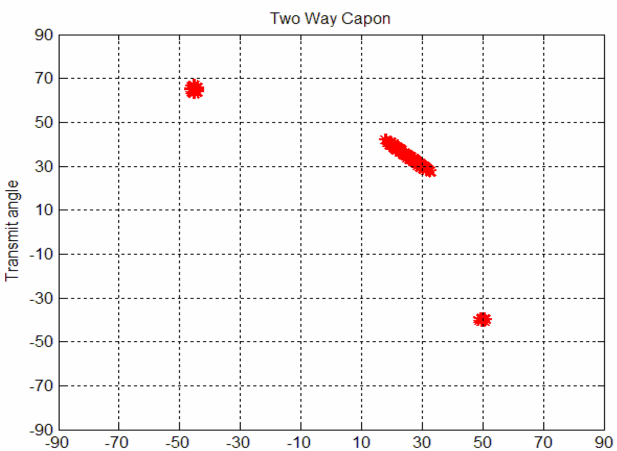
$$\max(M_t, M_r) \geq 3 \text{ and } M_t M_r - \min(M_t, M_r) \geq K$$

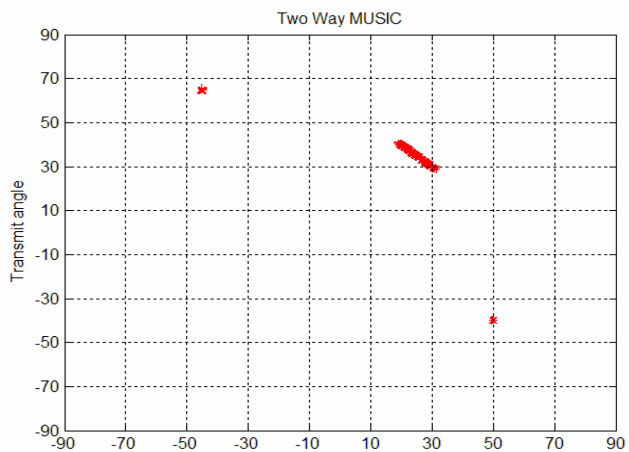
then uniqueness is guaranteed a.s. *[Jiang, Sidiropoulos, Ten Berge].*

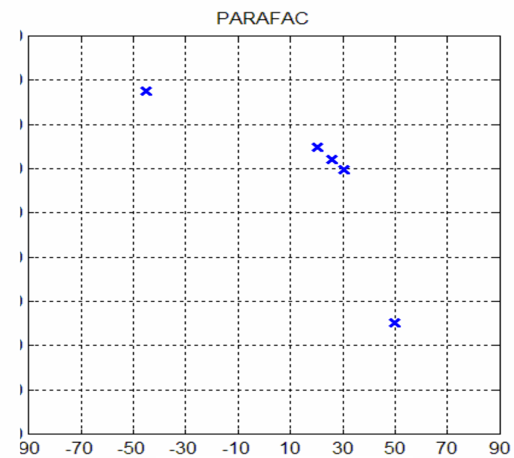
$M_t = M_r$	3	4	5	6	7	8
K_{max} condition 1	4	9	14	21	30	40
K_{max} condition 2	6	12	20	30	42	56

III. Localization via PARAFAC: simulations

$$K = 5, \{\theta_k\} = \{40°, 35°, 30°, -40°, 65°\}, \{\alpha_k\} = \{20°, 25°, 30°, 50°, -45°\}$$



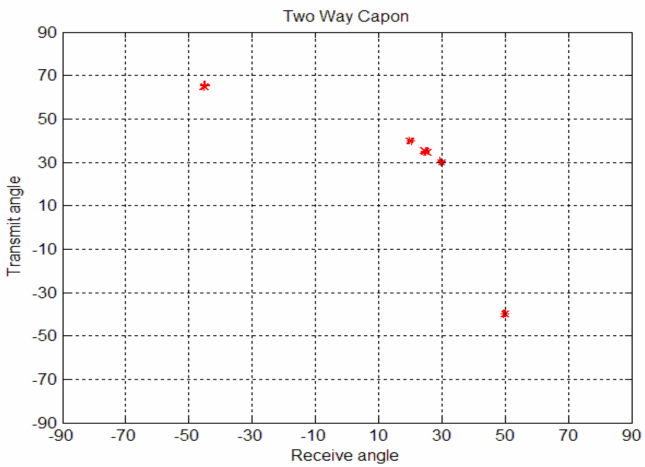


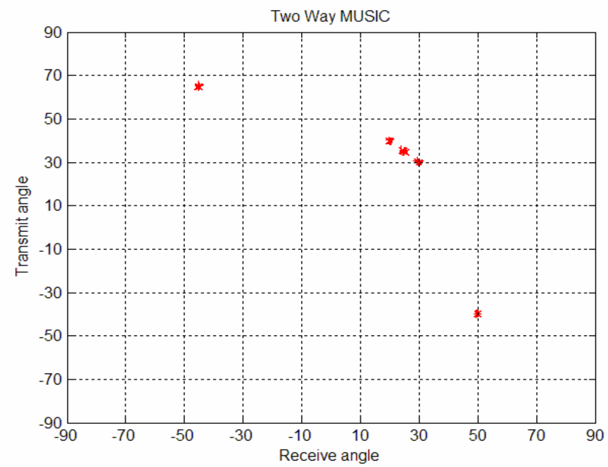


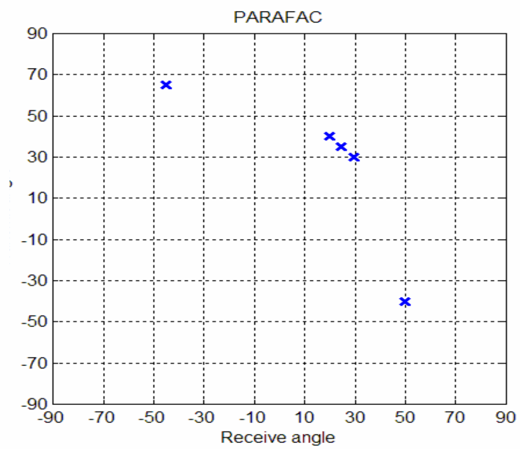
$M_t=M_r=4$ $M_t=M_r=4$ $M_t=M_r=4$

CAPON **MUSIC** **PARAFAC**

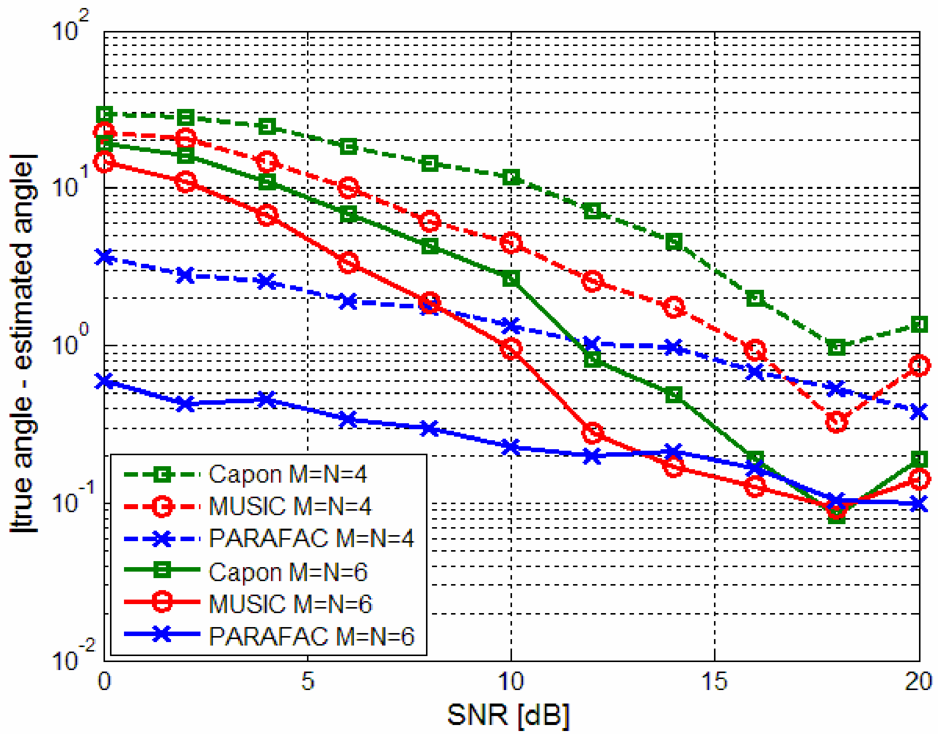
$M_t=M_r=9$ $M_t=M_r=9$ $M_t=M_r=9$







III. Localization via PARAFAC: simulations



K=4 targets,

$M_t = M_r = 4$ and $M_t = M_r = 6$,

100 Monte-Carlo runs

Angles randomly generated for each run (with minimum inter-target spacing of 5°)

IV. Conclusion

- PARAFAC = deterministic alternative to radar-imaging (Capon, MUSIC, etc)

➢ Guaranteed identifiability

➢ RCS fluctuations from pulse to pulse = time diversity
= 1 dimension of the observed tensor

- PARAFAC outperforms MUSIC and Capon

➢ Peak detection in radar-imaging fails for closely located targets

➢ PARAFAC = estimation based on exploitation of strong algebraic structure of observed data.

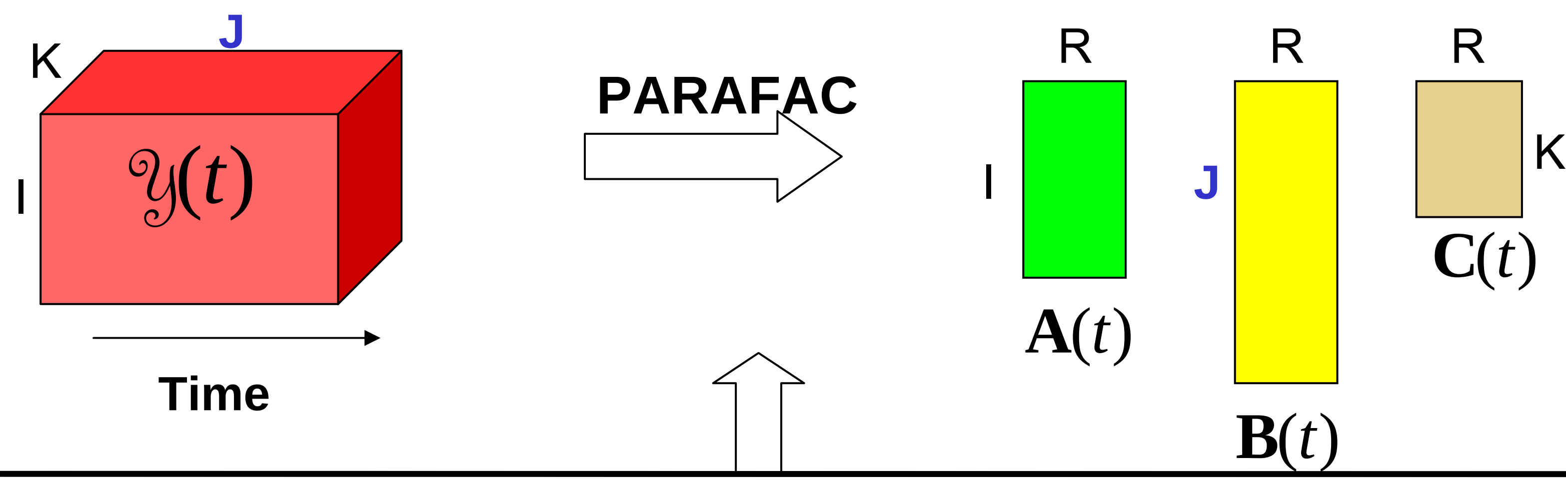
- Extension (work in progress):

Generalization to the case of multiple sufficiently spaced transmit and receive sub-arrays.

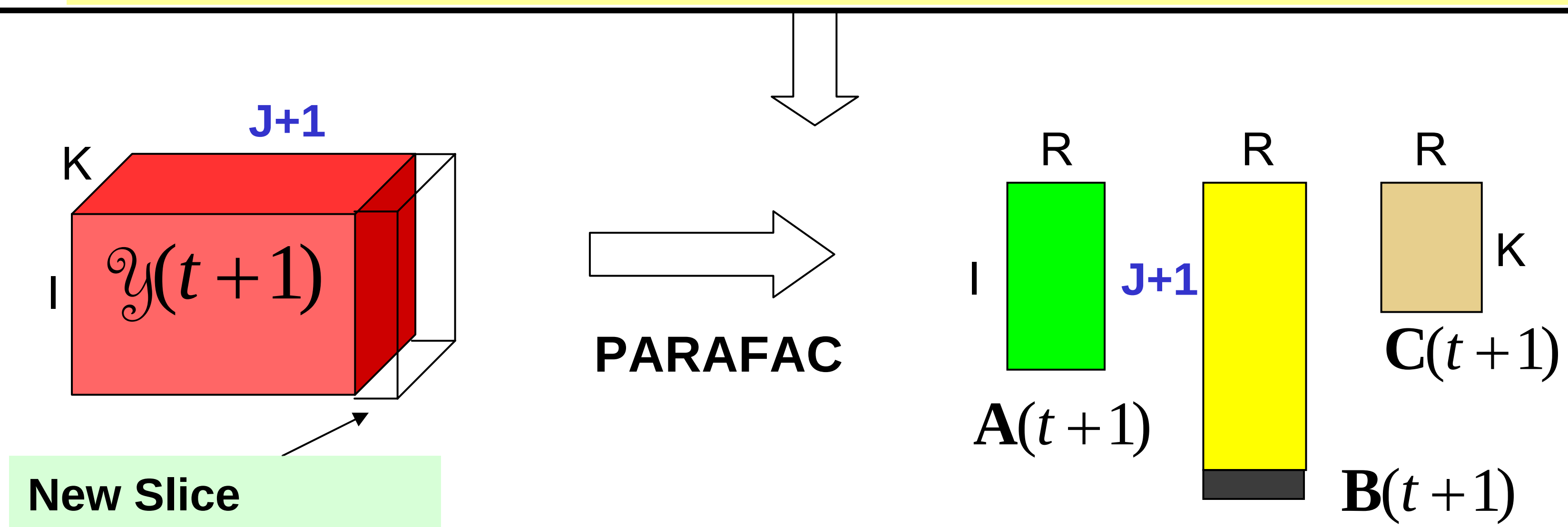
Appendix: Target tracking via adaptive PARAFAC

« Adaptive algorithms to track the PARAFAC decomposition »

[Nion & Sidiropoulos 2009]



LINK = adaptive algorithms to track the PARAFAC decomposition



Appendix: Target tracking via adaptive PARAFAC

5 moving targets. Estimated trajectories. Comparison between Batch PARAFAC (applied repeatedly) and PARAFAC-RLST (« Recursive Least Squares Tracking »)