A PARAFAC-based technique for detection and localization of multiple targets in MIMO radar systems

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Content of the talk

Context: MIMO radar system.

Problem: Detection and localization of multiple targets present in the same range-bin.

State of the art: Radar-imaging localization methods (e.g. Capon, MUSIC)

Limits: Radar-imaging fails for closely spaced targets + sensitivity to Radar Cross Section (RCS) fluctuations

Contribution: Novel method, deterministic, exploits multilinear algebraic structure of received data

\( \rightarrow \) PARAFAC Decomposition of an observed tensor
Roadmap

I. Introduction  (problem statement + data model)

II. State of the art  (Localization via Capon beamforming and MUSIC)

III. Localization via PARAFAC

IV. Conclusion and perspectives
I. Introduction: problem statement

- $K$ targets in the same range-bin
- Transmitter Tx and receiver Rx equipped with closely spaced antennas
- Target = a point source in the far field

Problem: estimate the number of targets and their DODs and DOAS
## I. Introduction: parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$M_t$</td>
<td>transmit colocated antennas</td>
</tr>
<tr>
<td>$M_r$</td>
<td>receive colocated antennas</td>
</tr>
<tr>
<td>$K$</td>
<td>targets in the range-bin of interest</td>
</tr>
<tr>
<td>$A(\theta) = [a(\theta_1), \ldots, a(\theta_K)]$</td>
<td>the $M_t \times K$ transmit steering matrix</td>
</tr>
<tr>
<td>$B(\alpha) = [b(\alpha_1), \ldots, b(\alpha_K)]$</td>
<td>the $M_r \times K$ receive steering matrix</td>
</tr>
<tr>
<td>$S = [s_1(t); s_2(t); \ldots; s_{M_t}(t)]$</td>
<td>is $M_t \times L$, holds $M_t$ mutually orthogonal transmitted pulse waveforms, with</td>
</tr>
<tr>
<td>$L$</td>
<td>samples per pulse period</td>
</tr>
<tr>
<td>$Q$</td>
<td>consecutive pulses are transmitted</td>
</tr>
<tr>
<td>$\beta_{kq}$</td>
<td>RCS reflection coeff. of target $k$ during pulse $q$</td>
</tr>
</tbody>
</table>
I. Introduction: data model

Assumption: Swerling case II target model

« Receive and Transmit steering matrices $B(\alpha)$ and $A(\theta)$ constant over the duration of $Q$ pulses while the target reflection coefficients $\beta_{kq}$ are varying independently from pulse to pulse ».

\[
X_q = B(\alpha) \text{diag}([\beta_{1q}, \ldots, \beta_{Kq}]) A^T(\theta)S + W_q, \quad q = 1, \ldots, Q
\]

$M_r \times L$ received data

$\rightarrow$ Times of arrival known (targets in the same range-bin).

$\rightarrow$ Right multiply by $(1/L)S^H$ and simplify $(1/L)SS^H = I$

\[
Y_q = B(\alpha) \sum_q A^T(\theta) + Z_q, \quad q = 1, \ldots, Q
\]

$M_t \times M_t$ received data after matched filtering
II. State of the art: single-pulse radar-imaging

Radar-imaging techniques working on per-pulse basis:

\[ X_q = B(\alpha)\Sigma_q A^T(\theta)S + W_q, \quad q = 1, \ldots, Q \]

→ Beamforming techniques [Xu, Li & Stoica].

Example: Capon Beamforming. Suppose colocated arrays (α=θ).

\[
\hat{\beta}(\theta) = \frac{b^H(\theta)R_{XX}^{-1}X_qS^H\a^*(\theta)}{L[b^H(\theta)R_{XX}^{-1}b(\theta)][a^T(\theta)\a^*(\theta)]}, \quad R_{XX} = \frac{1}{L}X_qX_q^H
\]

→ MUSIC estimator.

\[
P_{MUSIC}(\theta) = \frac{1}{b^H(\theta)E_wE^H_wb(\theta)}, \quad E_w = \text{noise eigenvectors of } R_{XX}
\]
II. State of the art: single-pulse radar-imaging

Typical Capon and MUSIC spectra for a given pulse

| Widely spaced targets (-30°, 10°, 40°) | Closely spaced targets (-30°, -25°, -20°) |

- **Problem 1:** single lobe occurs for closely located targets
- **Problem 2:** update spectrum for each new pulse → scintillation due to fading (fluctuations of RCS coeff. from pulse to pulse)
II. State of the art: multiple-pulses radar-imaging

Q: Mitigate RCS fluctuations? → first need a multi-pulse data model

\[ Y_q = B(\alpha) + \sum_{q} A^T(\theta) + Z_q \]

\[ y_q = [a(\theta_1) \otimes b(\alpha_1), ..., a(\theta_K) \otimes b(\alpha_K)] [\beta_{q1}, ..., \beta_{qK}]^T + z_q \]

\[ = A(\theta) \otimes B(\alpha) \]

\[ = c_q^T \]

\[ Y = [A(\theta) \otimes B(\alpha)] C^T + Z \]
II. State of the art: multiple-pulses radar-imaging

Radar-imaging techniques working on a multi-pulse basis:

\[
Y = [A(\theta) \otimes B(\alpha)] C^T
\]

- Capon beamforming \([Yan, Li, Liao]\)

\[
P_{\text{Capon}} (\theta, \alpha) = \frac{1}{(a(\theta) \otimes b(\alpha))^H R_{YY}^{-1} (a(\theta) \otimes b(\alpha))}
\]

- MUSIC

\[
P_{\text{MUSIC}} (\theta, \alpha) = \frac{1}{(a(\theta) \otimes b(\alpha))^H E_w E_w^H (a(\theta) \otimes b(\alpha))}
\]

\[E_w = \text{noise eigenvectors of } R_{YY}\]
II. State of the art: multiple-pulses radar-imaging

\[ K = 5, \{\theta_k\} = \{40^\circ, 35^\circ, 30^\circ, -40^\circ, 65^\circ\}, \{\alpha_k\} = \{20^\circ, 25^\circ, 30^\circ, 50^\circ, -45^\circ\} \]
III. Localization via PARAFAC: overview

Problems:

- Capon and MUSIC 2D-imaging work on multi-pulse basis but fail if no distinguishable lobe for each target (e.g. closely located targets)
- Capon and MUSIC spectra have to be computed for each pair of angles → time consuming for dense angular grid

→ Our contribution: starting from the same data model,

\[
Y = [A(\theta) \odot B(\alpha)] C^T
\]

exploitation of the algebraic structure of \(Y\) is sufficient for blind estimation of \(A(\theta), B(\alpha)\) and \(C\).

Indeed \(Y\) follows the well-known PARAFAC model.
III. Localization via PARAFAC: model

\[
Y_q = B(\alpha) \sum_q A^T(\theta) + Z_q
\]

\(M_r \times M_t\) matrix observed \(Q\) times, \(q=1,\ldots,Q\). \(B(\alpha)\) and \(A(\theta)\) fixed over \(Q\) pulses.

\(Q\times K\) matrix \(C\), \([C]_{qk} = \beta_{qk}\)

PARAFAC decomposition:
\(Y = \text{Sum of } K \text{ rank-1 tensors}\).
Each target contribution is a rank-1 tensor.
III. Localization via PARAFAC: summary

- Given the \((M_1 \times M_2 \times Q)\) tensor \(Y\), compute its PARAFAC decomposition in \(K\) terms to estimate \(A(\theta)\), \(B(\alpha)\) and \(C\).
  
  - Several algorithms in the literature (e.g. Alternating Least Squares (ALS), ALS+Enhanced Line Search, Levenberg-Marquardt, Simultaneous Diagonalization, …)

- **Key point:** under some conditions (next slide), PARAFAC is unique up to trivial indeterminacies:
  - Columns of \(A(\theta)\), \(B(\alpha)\) and \(C\) arbitrarily permuted (same permutation)
  - Columns of \(A(\theta)\), \(B(\alpha)\) and \(C\) arbitrarily scaled (scaling factor removed by recovering the known array manifold structure on the steering matrices estimates, after which the DODs and DOAs are extracted).
III. Localization via PARAFAC: uniqueness

- **Condition 1:** \( \mathbf{A}(\theta) \) and \( \mathbf{B}(\alpha) \) full rank and \( \mathbf{C} \) full-column rank. If
  
  \[
  K \geq 2 \text{ and } M_t(M_t - 1)M_r(M_r - 1) \geq 2K(K - 1)
  \]
  
  then uniqueness is guaranteed a.s. [De Lathauwer].

- **Condition 2:** \( \mathbf{A}(\theta) \) and \( \mathbf{B}(\alpha) \) are full rank Vandermonde matrices and \( \mathbf{C} \) full-column rank. If
  
  \[
  \max(M_t, M_r) \geq 3 \text{ and } M_tM_r - \min(M_t, M_r) \geq K
  \]
  
  then uniqueness is guaranteed a.s. [Jiang, Sidiroopoulos, Ten Berge].

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_t=M_r )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>( K_{\text{max}} )</td>
<td></td>
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</tr>
<tr>
<td>condition 1</td>
<td>4</td>
<td>9</td>
<td>14</td>
<td>21</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>condition 2</td>
<td>6</td>
<td>12</td>
<td>20</td>
<td>30</td>
<td>42</td>
<td>56</td>
</tr>
</tbody>
</table>
III. Localization via PARAFAC: simulations

\[ K = 5, \{\theta_k\} = \{40^\circ, 35^\circ, 30^\circ, -40^\circ, 65^\circ\}, \{\alpha_k\} = \{20^\circ, 25^\circ, 30^\circ, 50^\circ, -45^\circ\} \]
III. Localization via PARAFAC: simulations

K=4 targets,
M_t = M_r = 4 and M_t = M_r = 6,
100 Monte-Carlo runs

Angles randomly generated for each run (with minimum inter-target spacing of 5°)
IV. Conclusion

- PARAFAC = deterministic alternative to radar-imaging (Capon, MUSIC, etc)
  - Guaranteed identifiability
  - RCS fluctuations from pulse to pulse = time diversity
    = 1 dimension of the observed tensor
- PARAFAC outperforms MUSIC and Capon
  - Peak detection in radar-imaging fails for closely located targets
  - PARAFAC = estimation based on exploitation of strong algebraic structure of observed data.

- Extension (work in progress):
  Generalization to the case of multiple sufficiently spaced transmit and receive sub-arrays.
Appendix: Target tracking via adaptive PARAFAC

« Adaptive algorithms to track the PARAFAC decomposition »
[Nion & Sidiropoulos 2009]
Appendix: Target tracking via adaptive PARAFAC

5 moving targets. Estimated trajectories. Comparison between Batch PARAFAC (applied repeatedly) and PARAFAC-RLST (« Recursive Least Squares Tracking »)