Levenberg-Marquardt Computation of the Block Factor Model for Blind Multi-User Access in Wireless Communications

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Keywords

- Research Area: Blind Source Separation (BSS)
- Application: Wireless Communications (DS-CDMA system here)
- Constraints: Multiuser system, multipath propagation, Inter Symbol Interference (ISI), Gaussian Noise
- Assumptions: No knowledge of the channel, neither of CDMA codes, noise level and antenna array response (BLIND approach)
- Objective: Separate each user's contribution and estimate information symbols
- Method:
  - Deterministic: relies on multilinear algebra
  - How? store observations in a third order tensor and decompose it in a sum of users' contributions
- Power:
  - No orthogonality constraints between factors
  - Tensor Model « richer » than matrix model

Plan

Introduction

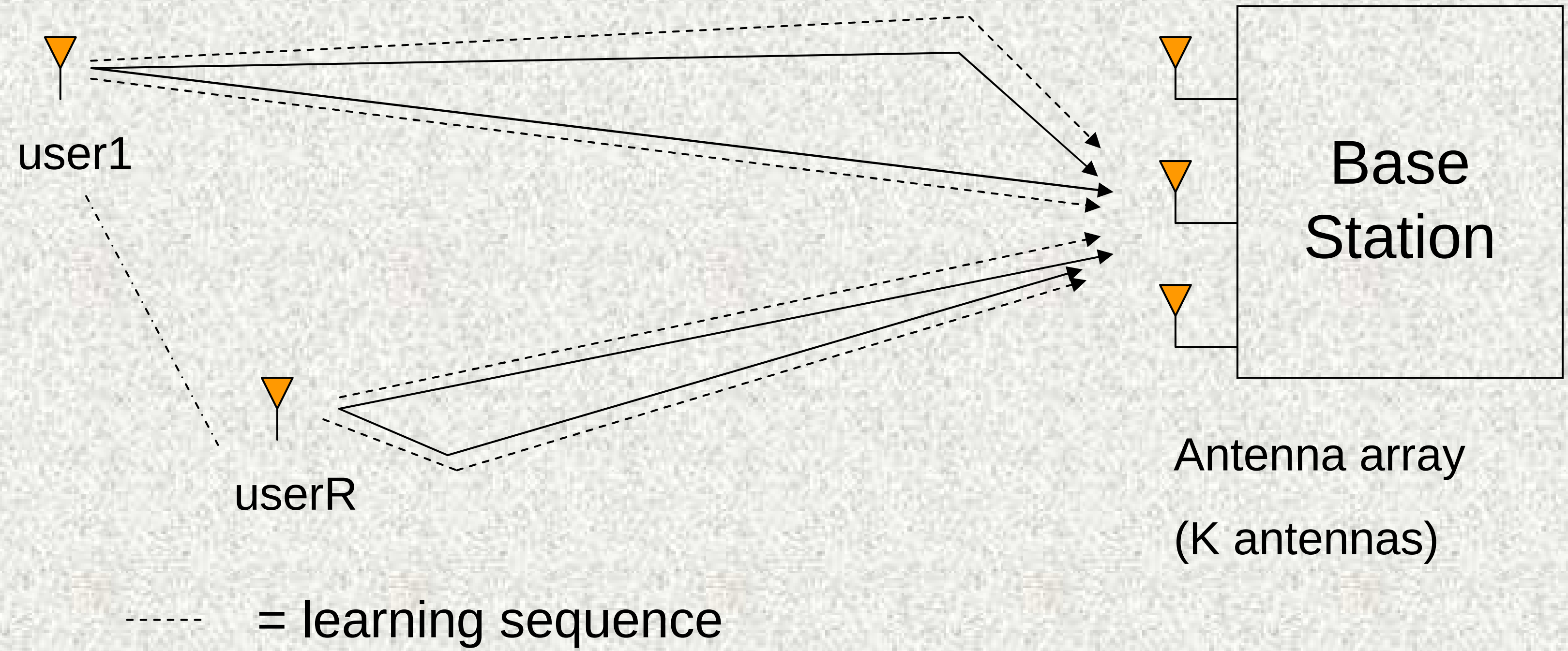
1. PARAFAC Decomposition
   - 1.1 Concept
   - 1.2 Uniqueness of the decomposition
   - 1.3 Application: direct path propagation
   - 1.4 Algorithm: Standard ALS

2. Block Factor Model (BFM) Decomposition
   - 2.1 Problem: multipath propagation with ISI
   - 2.2 Received Signals: Analytic and algebraic forms
   - 2.2 Uniqueness of the Decomposition
   - 2.4 Algorithms: ALS vs. Levenberg-Marquardt

3. Simulation Results

Conclusion

Overview of a wireless communication system



➢ The R users transmit at the same time within the same bandwidth towards the antenna array.

➢ We want to estimate their signals without knowledge of the learning seq. (i.e. BLIND estimation)

Blind Signal Separation: Why?

Several motivations among others:

- Elimination or reduction of the learning frames: more than 40 % of the transmission rate devoted to training in UMTS
- Training not efficient in case of severe multipath fading or fast time varying channels
- Applications: eavesdropping, source localization, …
- If learning seq. is unavailable or partially received

Blind Signal Separation: How?

Overview of usual techniques

- Usual formulation: **X = H . S** (matrix decomposition)

X : observation matrix

H : channel matrix
S : source matrix
} Unknown

- How identify **S** ?

- Temporal prop. (FA, CM, …)
- Statistical prop. (HOS, ICA, …)
- Spatial prop. (array geometry)
  - → estimate DOA's (ESPRIT, MUSIC)
  - → extract signal

Blind Signal Separation: How?

Our approach: Tensor decomposition

Exploit 3 available diversities:

- Antenna array → Spatial Diversity
- Collect samples (J.Ts) → Temporal Diversity
- Temporal over-sampling (at the chip rate) → Spectral Diversity
- The methods we develop can be applied in systems where 3 diversities are available (e.g. MIMO CDMA)

Build a 3rd order Tensor with the observations:

The original data will be estimated by means of:

- Standard PARAFAC (PARAllel FACtor) decomposition
- Block Factor Model (BFM) decomposition

1. PARAFAC Decomposition

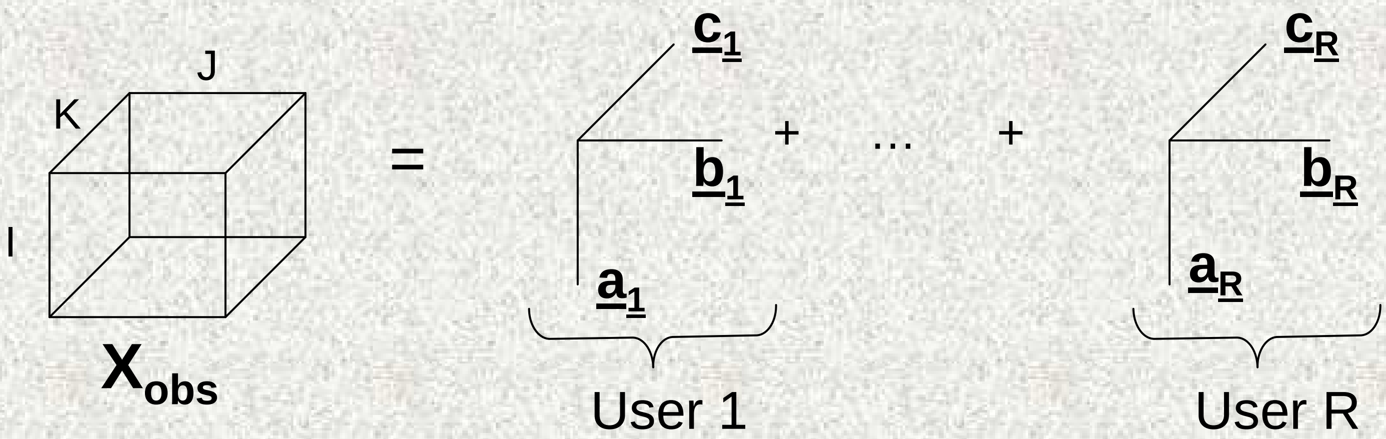
(PARAllel FACtor analysis)

(Harshman 1970 , Bro 1997, Sidiropoulos 2000)

→ Direct Path Propagation

Concept

Well-known method: decompose the tensor of observations in a sum of a minimum of rank-1 terms



Each user contribution is a rank-1 tensor, i.e. built from 3 loading vectors

Constraint: Uniqueness of the decomposition

Loading Matrices
$$\begin{cases} A = [a_1 \ldots a_R] & \in C^{I\times R} \\ B = [b_1 \ldots b_R] & \in C^{J\times R} \\ C = [c_1 \ldots c_R] & \in C^{K\times R} \end{cases}$$

PARAFAC decomposition unique if (sufficient condition):

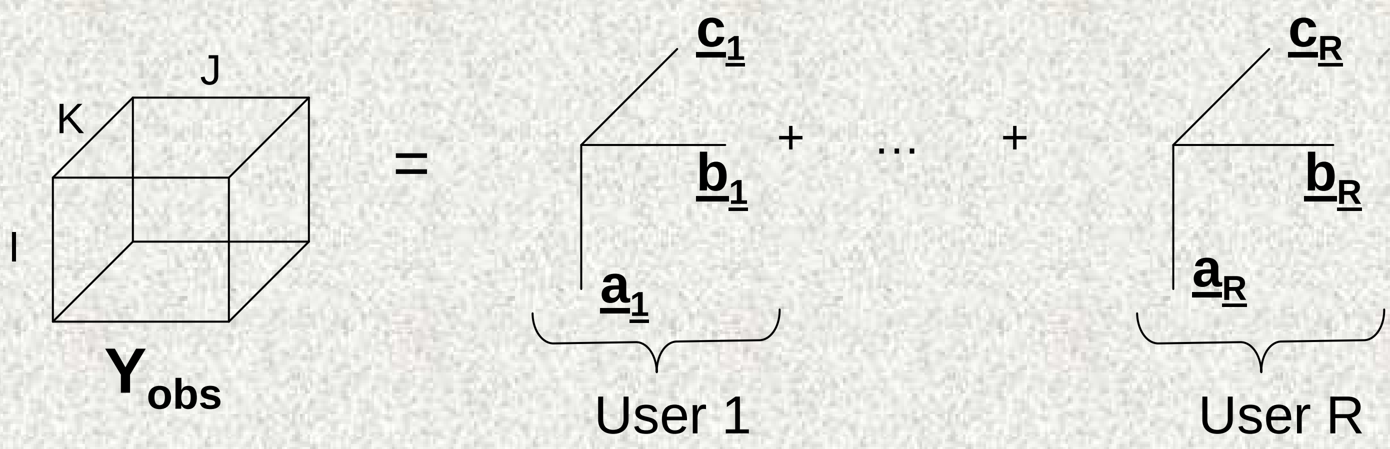
k(A)+k(B)+k(C) ≥ 2(R+1)

(k:Kruskal rank)

- Bound on the max. number of users R
- No orthogonality constraints on loading matrices

PARAFAC

Application: direct-path only propagation (Sidiropoulos et al.,2000)



For the r^{th} user:

a_r contains the I chips of the CDMA code

b_r contains the J symbols successively emitted

c_r contains the response of the K antennas

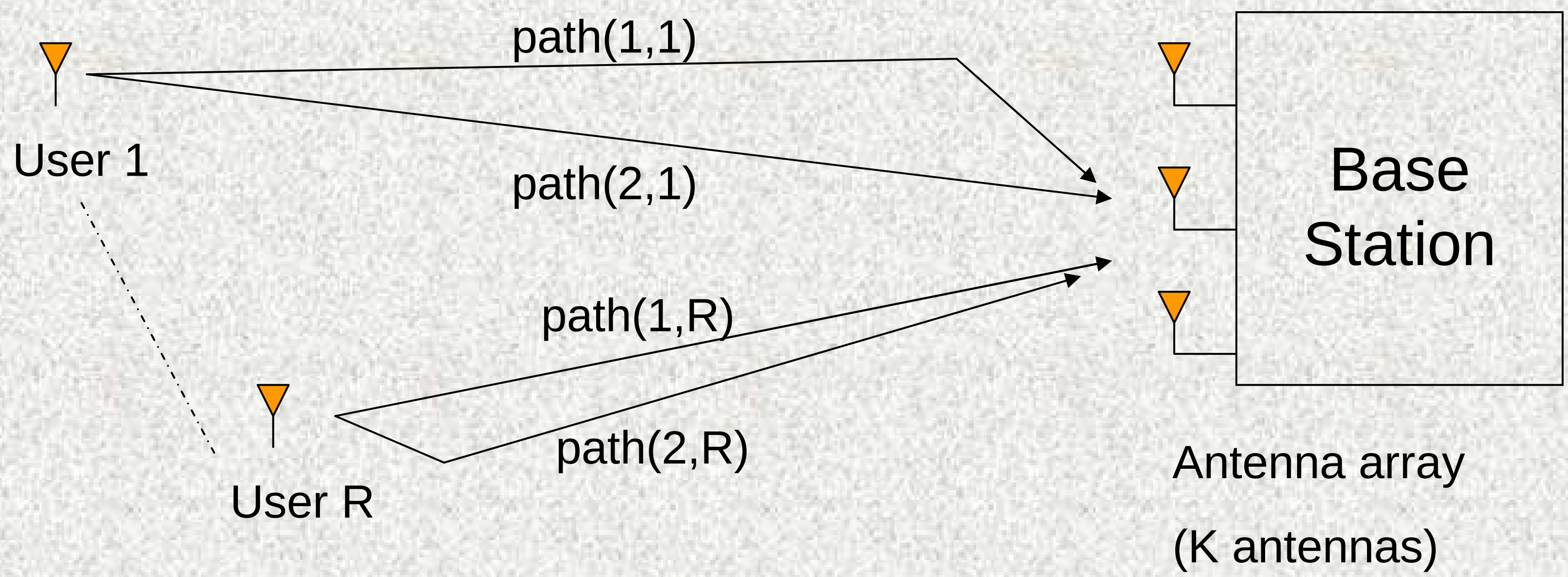
2. Block Factor Model (BFM) decomposition

(A New Tensor Decomposition that generalizes PARAFAC)

(Nion and De Lathauwer, ICASSP 2006, SPAWC 2006)

➡ Uplink CDMA, Multipath Propagation with ISI

Propagation model: Multipath



- One path = One angle of Arrival = One channel modeled by FIR filter.
- We assume P paths per user.
- Memory of the Channel → ISI. We assume L interfering symbols per user.

Received Signal (analytic expression)

$$x_{ijk} = \sum_{r=1}^{R} \sum_{p=1}^{P} a_k(\theta_{rp}) \sum_{l=1}^{L} h_{rp}(i + (l-1)I) s_{j-l+1}^{(r)}$$

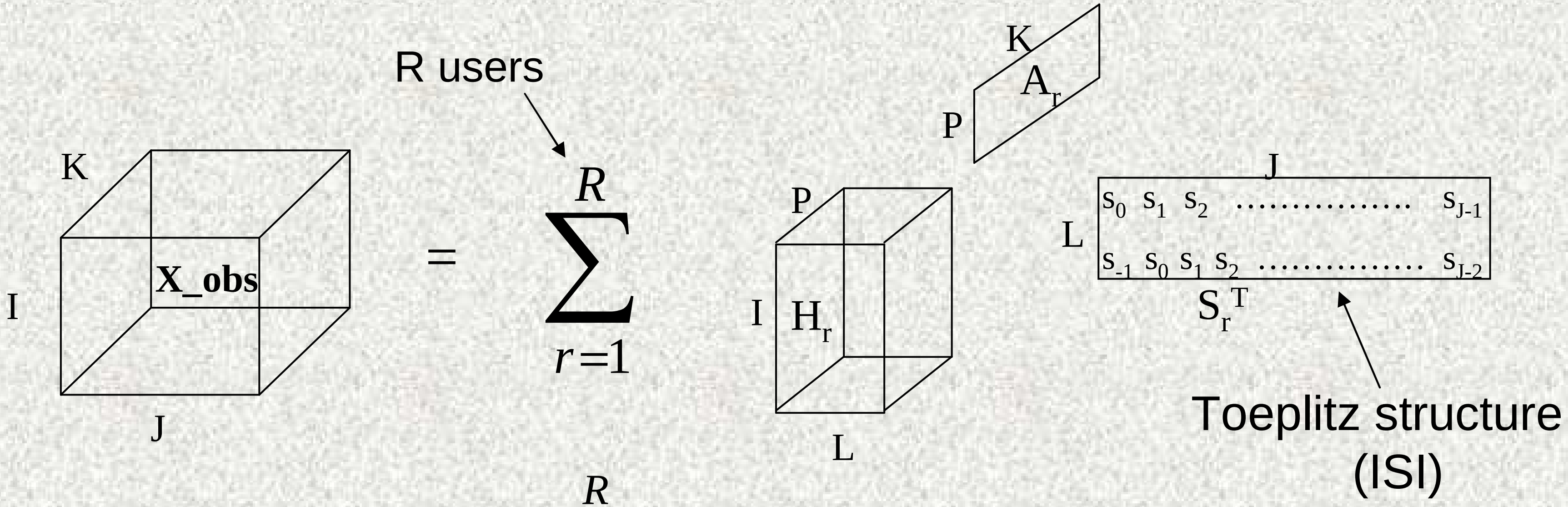
Contribution of R users

Contribution of P paths

Contribution of L interfering symbols

- x_{ijk} : i^{th} sample (chip) within the j^{th} symbol period of the overall signal received by the k^{th} antenna
- $a_k(\theta_{rp})$: response of the k^{th} antenna to p^{th} path incoming from the r^{th} user (angle of arrival θ_{rp})
- h_{rp} : convolution of the impulse response of the p^{th} channel with the CDMA code of the r^{th} utilisateur
- $s^{(r)}_{j-l+1}$: symbol transmitted by the r^{th} user at time $(j-l+1)T_s$

Received Signal (algebraic form):BFM



$$= \sum_{r=1}^{R} H_r \times_2 S_r \times_3 A_r$$

R: nb. of users	L: nb. of interfering symbols
I: length of CDMA code	P: nb. of paths
J: nb. of symbols collected	K: nb. of antennas

Uniqueness of the BFM decomposition

Sufficient condition for identifiability: (De Lathauwer 2005)

$$\min\left(\left\lfloor \frac{I}{\max(L,P)} \right\rfloor, R\right) + \min\left(\left\lfloor \frac{J}{L} \right\rfloor, R\right) + \min\left(\left\lfloor \frac{K}{P} \right\rfloor, R\right) \geq 2R+2$$

Oversampling factor → I

Nb of antennas → K

Nb. Of paths → P

Max users → R max

I	J	K	L	P	**R max**
4	30	4	2	2	2
6	30	6	2	2	4
16	30	4	3	2	5

Nb. of symbols → J

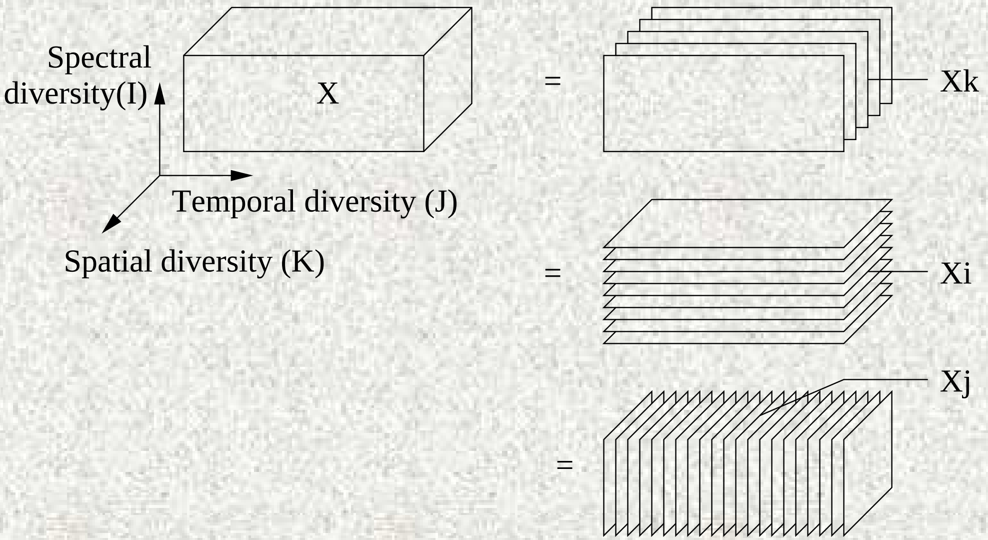
Nb. Of ISI → L

⇒ Identifiability guaranteed even with more users (R=5) than antennas (K=4)

Computation of the BFM decomposition (1)

➢ Objective: minimize $\Phi = \|X - X^{(n)}\|^2$, with $X^{(n)}$ built from $A^{(n)}$,$H^{(n)}$ and $S^{(n)}$

➢ ALS (Alternating Least Squares) algorithm



$$\mathbf{M}_{I\times KJ} = cat[\mathbf{X}_k]$$

$$\mathbf{M}_{J\times IK} = cat[\mathbf{X}_i]$$

$$\mathbf{M}_{K\times JI} = cat[\mathbf{X}_j]$$

- Alternate update of unknown factors in the LS sense

$$\begin{cases} \mathbf{S}_r^{(n)} \text{ from } \mathbf{H}_r^{(n-1)},\ \mathbf{A}_r^{(n-1)} \text{ and } \mathbf{M}_{I\times KJ} \\ \mathbf{H}_r^{(n)} \text{ from } \mathbf{A}_r^{(n-1)},\ \mathbf{S}_r^{(n)} \text{ and } \mathbf{M}_{J\times IK} \\ \mathbf{A}_r^{(n)} \text{ from } \mathbf{S}_r^{(n)},\ \mathbf{H}_r^{(n)} \text{ and } \mathbf{M}_{K\times JI} \end{cases}$$

Computation of the BFM decomposition (2)

➢ <u>Objective:</u> minimize $\Phi=||X-X^{(n)}||^2$, with $X^{(n)}$ built from $A^{(n)}$,$H^{(n)}$ and $S^{(n)}$

➢ LM (Levenberg Marquardt) algorithm = « damped Gauss-Newton »

- Concatenate all unknowns in a vector **p**
- $\Phi = || X-X^{(n)} ||^2 = || \mathbf{r}(\mathbf{p}) ||^2$ (**r** = mapping: **p** → **r**(**p**) **,** vector of residuals)
- Find update of **p** by solving modified G.N. normal eq :

$$(J^T J + \lambda I)\Delta p^{(n)} = -g$$

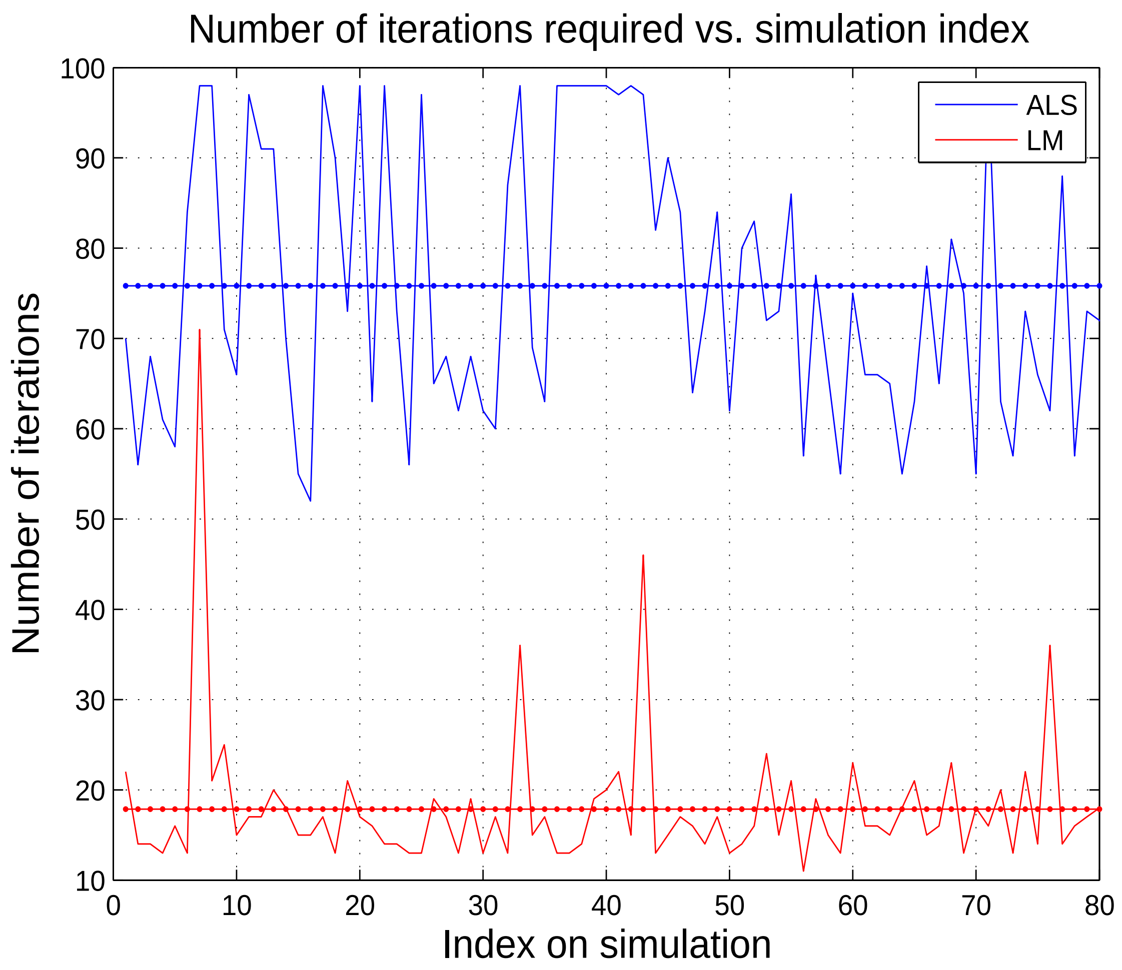
- gradient: $\mathbf{g} = \frac{\partial \Phi}{\partial \mathbf{p}}$ Jacobian: $j_{mf} = \frac{\partial \mathbf{r}_m(\mathbf{p})}{\partial \mathbf{p}_f}$

- damping factor: λ is increased until J^TJ is full-rank

Results of simulations: Noise-free case (1)

No Noise: global minimum of $\Phi=||X-X^{(n)}||^2$ is 0

Fig: Nb of iterations for each of 80 simulations



Parameters:

[I J K L P R] =

[16 30 4 3 2 5]

Mean nb. of iter:

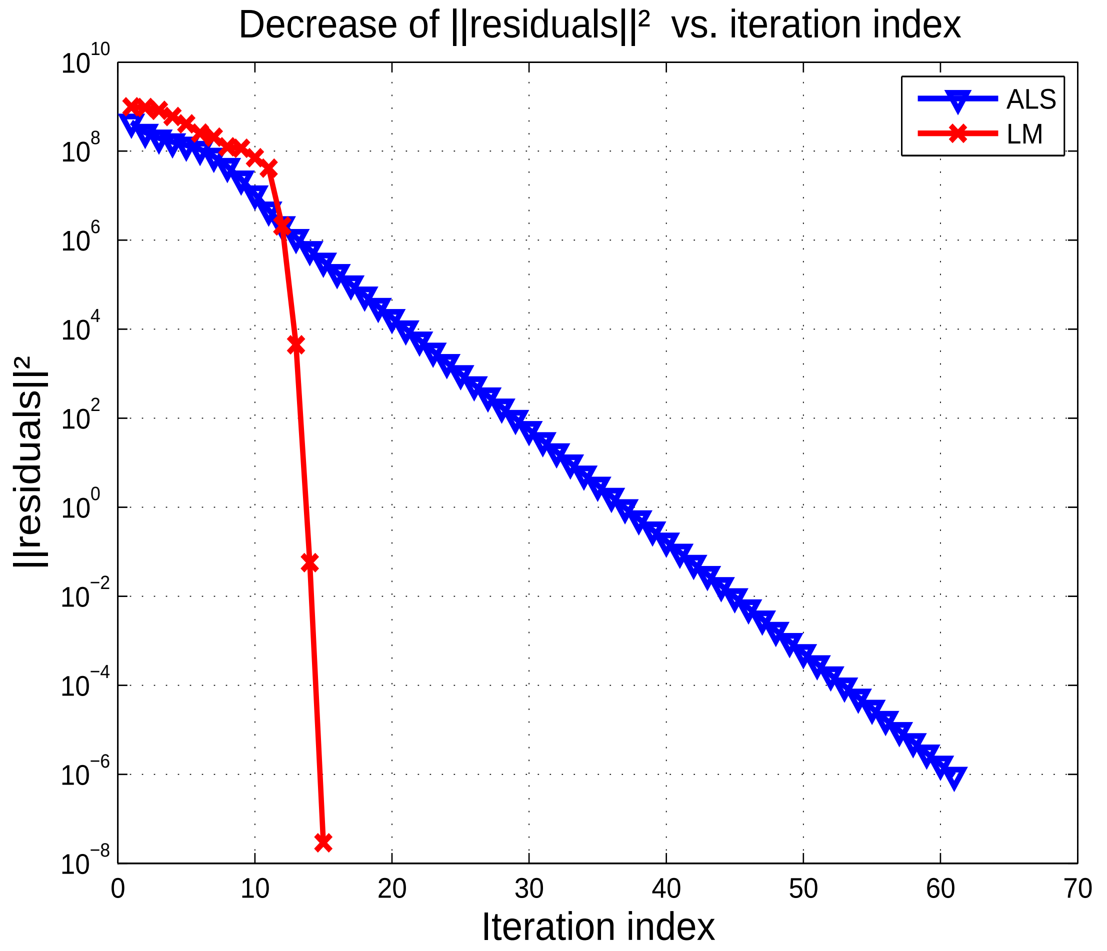
ALS : 76

LM : 18

Results of simulations: Noise-free case (2)

No Noise: global minimum of $\Phi=||X-X^{(n)}||^2$ is 0

Fig: Evolution of Φ vs. iteration Index (1 simulation)



Parameters:

[I J K L P R] =

[16 30 4 3 2 5]

Stop crit. : $\Phi < 10^{-6}$

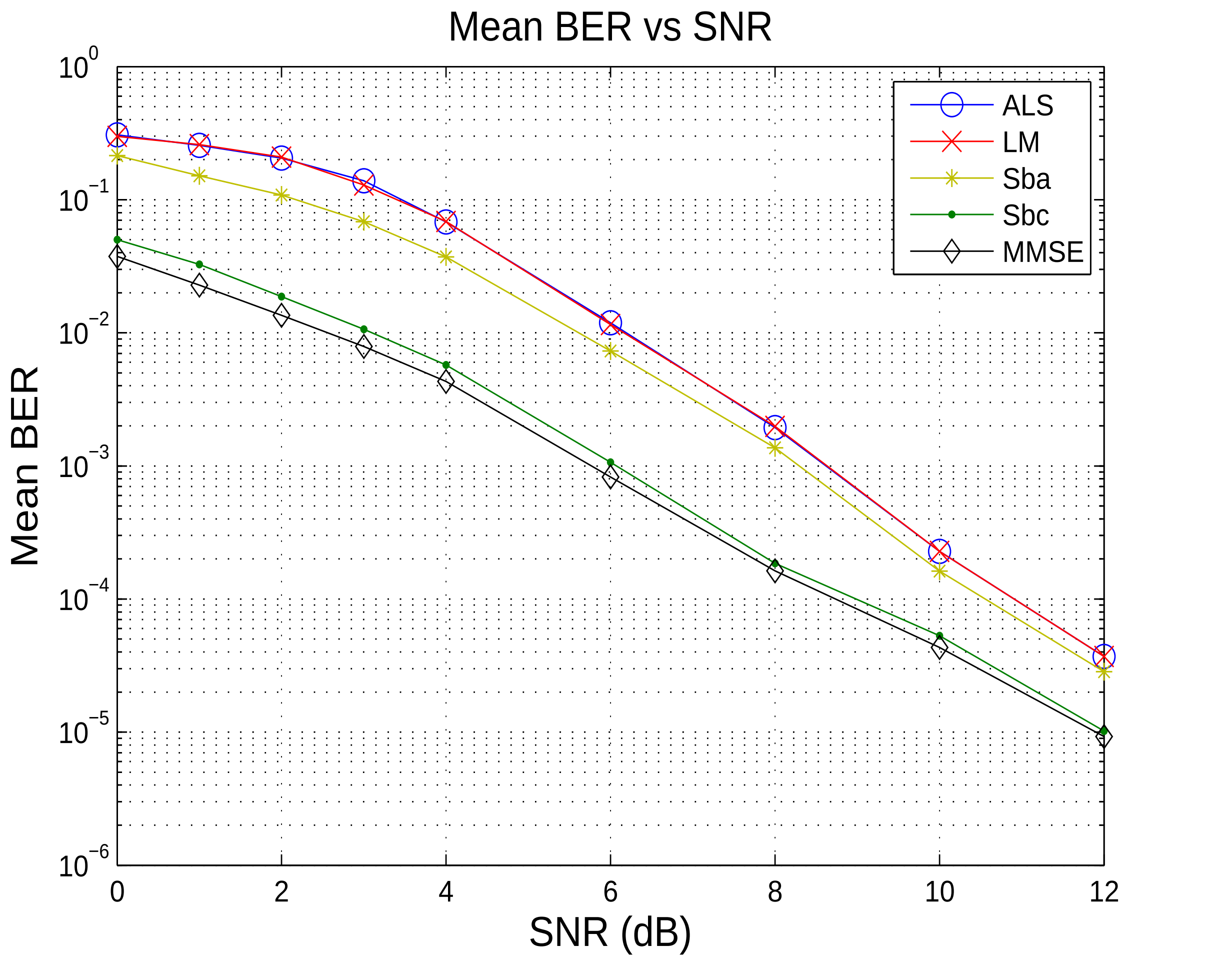
ALS : 61 iter.
LM : 15 iter.

LM: gradient steps then GN steps

Results of Monte Carlo simulations (1)

AWGN: BER vs. SNR (Blind, Semi-Blind & Non-Blind)

Fig: Mean BER vs. SNR (1000 MC runs)



Parameters:

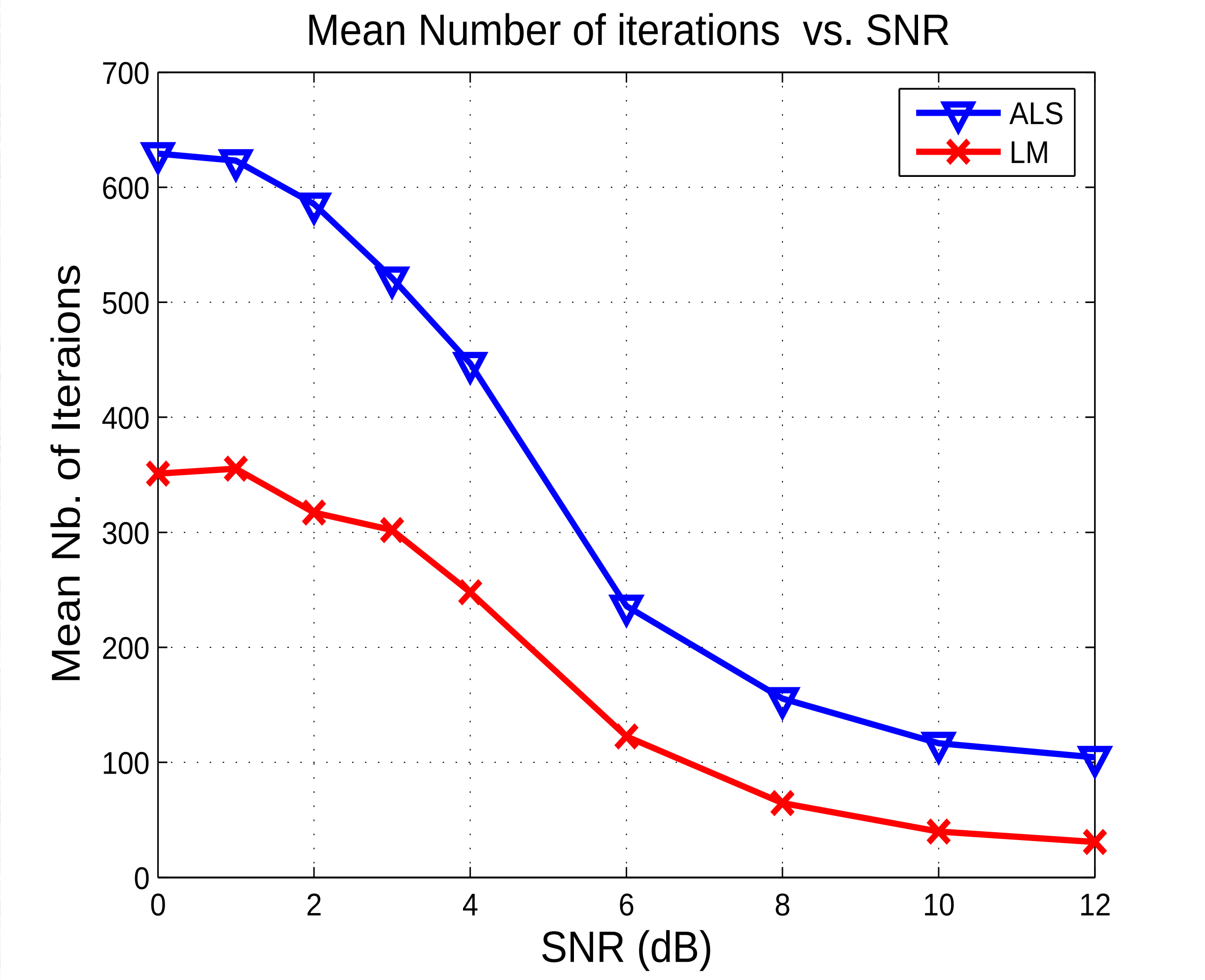
[I J K L P R] =

[16 30 4 3 2 5]

Results of Monte Carlo simulations (2)

AWGN: BER vs. SNR (Blind, Semi-Blind & Non-Blind)

Fig: Mean nb. of iter. vs. SNR (1000 MC runs)



Parameters:

[I J K L P R] =

[16 30 4 3 2 5]

Conclusion

- PARAFAC : Well-known model (since 70's)
  - Tensor Decomposition in terms of rank-1
  - Blind receiver for direct-path propagation

- BFM (Block Factor Model):
  - Generalization of PARAFAC
  - Powerful blind receiver for multi-path propagation with ISI
  - Weak assumptions: no orthogonality constraints, no independence between sources, no knowledge on CDMA code, neither of antenna response and Channel.
  - Fundamental Result: Uniqueness of the decomp. to guarantee identifiability
  - Performances close to non-blind MMSE
  - Algorithms: LM faster than ALS in terms of iter.