LINE SEARCH COMPUTATION OF THE BLOCK FACTOR MODEL FOR BLIND MULTI-USER ACCESS IN WIRELESS COMMUNICATIONS

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We present a technique for the computation of the Block Factor Model, which has been introduced in [1] as a powerful blind receiver for DS-CDMA signals received on an antenna array, in the context of multi-path propagation with Inter Symbol Interference (ISI).
This receiver relies on a new third-order tensor decomposition which is computed by an ALS algorithm improved by a Line Search scheme.

Communication System

- Blind Signal Separation: Why? Estimation of the data relative to each user without prior knowledge of the learning sequence.
  - Get higher communication rate.
  - Eavesdropping.
  - Source localisation.
  - Case of learning sequence unavailable or partially received.

- Parameters and propagation model:
  - R: Nb of users, transmitting at the same time within the same bandwidth.
  - I: Spreading Factor of CDMA codes.
  - J: Duration of the observation window (in Symbol Periods).
  - $I \times J$ samples collected at the receiver.
  - K: Nb of receiving Antennas.
  - P: Nb of reflected paths per user (Multipath Propagation).
  - L: Nb of interferring symbols (Inter Symbol Interference, ISI).

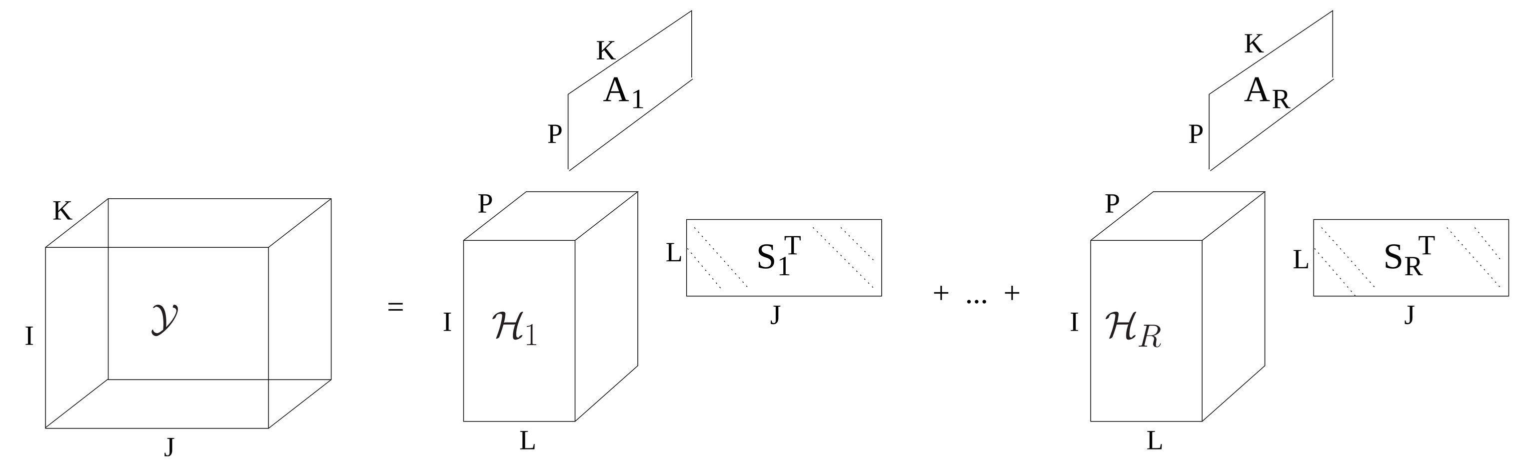
- Chip-Rate Sampled Received Signal: Analytic Form

Antenna Response — Channel Effect — Transmitted Symbol

$$y_{ijk} = \sum_{r=1}^{R} \sum_{p=1}^{P} a_k(\theta_{rp}) \sum_{l=1}^{L} h_{rp}(i + (l-1)I)\, s_{j-l+1}^{(r)}$$

Contribution of R users — Contribution of P paths — Contribution of L ISI

- Chip-Rate Sampled Received Signal: Algebraic Form



Block Factor Model (BFM)

- The Problem consists of the decomposition of the observation tensor in a sum of R terms.
- Each Term contains the information related to one particular user (channel, antenna response and symbols).
- The Toeplitz structure of each $\mathbf{S}_r$ is exploited.

Uniqueness of the Decomposition

- If BFM unique (up to some trivial indeterminacies): separation of the different user signals and estimation of the transmitted sequences are possible.
- Sufficient condition for uniqueness:

$$min\left(\left\lfloor\frac{J}{L}\right\rfloor, R\right) + min\left(\left\lfloor\frac{K}{P}\right\rfloor, R\right) + min\left(\left\lfloor\frac{I}{max(L,P)}\right\rfloor, R\right) \geq 2R+2, \quad (1)$$

- Upper bound on the number of users = maximal value of R that satisfies this equation. Example: $I = 12$, $J = 50$, $K = 4$, $L = 2$, $P = 2$, then $R = 6$ users can be allowed so more users than antennas is possible.

Line Search Computation of the Decomposition

- Objective: Given only $\mathcal{Y}$, estimate $\mathcal{H}_r$, $\mathbf{S}_r$ and $\mathbf{A}_r$ for each user.
- Matrix Representation of the unknowns: $\mathbf{A}$ $(K \times RP)$, $\mathbf{H}$ $(RLP \times I)$, $\mathbf{S}$ $(J \times RL)$.
- Optimization Problem to Solve: Minimize the cost function

$$\phi_{ALS} = \left\|\mathcal{Y} - \mathcal{Y}^{(n)}\right\|^2 = \left\|\mathbf{Y}^{(JK\times I)} - (\mathbf{S}^{(n)} \odot_R \mathbf{A}^{(n)})\mathbf{H}^{(n)}\right\|^2, \quad (2)$$

where the superscript n denotes the estimation at the n^{th} iteration.

- Previous Algorithm: Alternating Least Squares Algorithm (ALS)
Exploit the multilinearity of the model to alternate between conditional least-squares updates of $\mathbf{A}$, $\mathbf{S}$ and $\mathbf{H}$.
- Improvement scheme: Iterative Line Search + ALS (ILS + ALS)
Idea: Perform Linear Regression to predict the factors a number of iterations ahead before each ALS iteration.

$$\begin{cases} \mathbf{A}^{(new)} = \mathbf{A}^{(n-2)} + \rho\left(\mathbf{A}^{(n-1)} - \mathbf{A}^{(n-2)}\right) \\ \mathbf{S}^{(new)} = \mathbf{S}^{(n-2)} + \rho\left(\mathbf{S}^{(n-1)} - \mathbf{S}^{(n-2)}\right) \\ \mathbf{H}^{(new)} = \mathbf{H}^{(n-2)} + \rho\left(\mathbf{H}^{(n-1)} - \mathbf{H}^{(n-2)}\right) \end{cases}, \quad (3)$$

Then give $\mathbf{A}^{(new)}$, $\mathbf{S}^{(new)}$ and $\mathbf{H}^{(new)}$ instead of $\mathbf{A}^{(n-1)}$, $\mathbf{S}^{(n-1)}$ and $\mathbf{H}^{(n-1)}$ as inputs of ALS.

- Problem: Find the optimal value of $\rho \in \mathbb{C}$ that minimizes

$$\begin{aligned} \phi_{ILS}^{(n)} &= \left\|\left(\mathbf{S}^{(new)} \odot_R \mathbf{A}^{(new)}\right)\mathbf{H}^{(new)} - \mathbf{Y}^{(JK\times I)}\right\|^2 \\ &= \mathbf{u}^H \Delta \mathbf{u}. \end{aligned} \quad (4)$$

where $\mathbf{u} = \left[\,\rho^3\ \rho^2\ \rho\ 1\,\right]^T$ and Δ is a 4×4 known Hermitian matrix.

- Solution: Pose $\rho = r.e^{i\theta}$ and minimize $\phi_{ILS}^{(n)}$ alternately w.r.t. r and θ.

Iterative Line Search Scheme:

1. Partial minimization of $\phi_{ILS}^{(n)}$

(i) w.r.t. r: $\frac{\delta\phi_{ILS}^{(n)}(r)}{\delta r} = \sum_{p=0}^{5} c_p r^p$ **(ii)** w.r.t. θ (pose $t = tan(\frac{\theta}{2})$): $\frac{\delta\phi_{ILS}^{(n)}(t)}{\delta t} = \frac{\sum_{p=0}^{6} d_p t^p}{(1+t^2)^3}$

2. Repeat steps (i) and (ii) until $\left\|\phi_{ILS}^{(n)} - \phi_{ILS}^{(n-1)}\right\|^2 < \eta \quad (e.g.\ \ \eta = 10^{-1})$

This scheme is inserted in the standard ALS algorithm scheme as follows:

Summary of the ALS + ILS algorithm:

1- Initialize $\mathbf{A}^{(n-2)}$, $\mathbf{S}^{(n-2)}$, $\mathbf{H}^{(n-2)}$, $\mathbf{A}^{(n-1)}$, $\mathbf{S}^{(n-1)}$, $\mathbf{H}^{(n-1)}$, $n = 2$.

2- ILS Scheme:

- Find the optimal value of ρ from (i) and (ii).

- Build $\mathbf{A}^{(new)}$, $\mathbf{S}^{(new)}$ and $\mathbf{H}^{(new)}$ from (3).

3- ALS Steps:

- Find $\mathbf{A}^{(n)}$ from $\mathbf{S}^{(new)}$ and $\mathbf{H}^{(new)}$.

- Find $\mathbf{S}^{(n)}$ from $\mathbf{A}^{(n)}$ and $\mathbf{H}^{(new)}$.

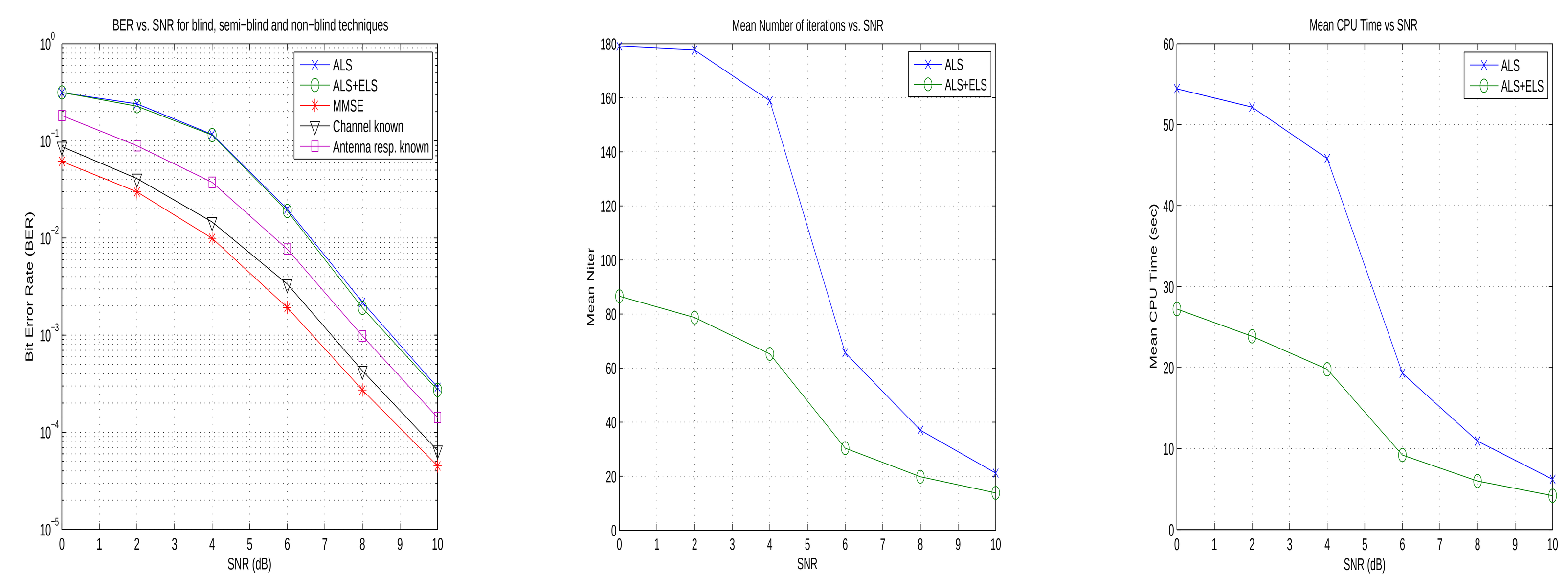
- Find $\mathbf{H}^{(n)}$ from $\mathbf{A}^{(n)}$ and $\mathbf{S}^{(n)}$.

4- Repeat from 2 until $c(n) < \epsilon$ (e.g. $\epsilon = 10^{-5}$), where $c(n) = \left\|\mathcal{Y}^{(n)} - \mathcal{Y}^{(n-1)}\right\|^2$.

- Increase n to $n+1$

Experimental Results

- Performance in presence of AWGN. Noisy tensor of observation: $\mathcal{Y}_{obs} = \mathcal{Y} + \mathcal{N}$.
- Parameters: $I = K = 6$, $J = 30$ QPSK symbols, $L = P = 2$, $R = 4$ (On the uniqueness bound).
- Comparison between performance of BFM Blind Receiver, MMSE (Non-Blind) Receiver, and Semi-Blind Receivers (either $\mathbf{H}$ or $\mathbf{A}$ known).



Conclusion

The Block Factor Model is a powerful blind receiver for multi-user access in wireless communications, with performance close to the MMSE receiver. The Iterative Line Search scheme greatly improves the convergence speed of the standard ALS algorithm [1]. Another faster algorithm has been developed in [2].

[1] Dimitri Nion and Lieven De Lathauwer, "A Block Factor Analysis based Receiver for Blind Multi-User Access in Wireless Communications", ICASSP 2006, May, Toulouse, France.

[2] Dimitri Nion and Lieven De Lathauwer, "Levenberg-Marquardt Computation of the Block Factor Model for Blind Multi-User Access in Wireless Communications", EUSIPCO 2006, September 4-8, Florence, Italy, accepted.

[3] Myriam Rajih and Pierre Comon, "Enhanced Line Search: A novel Method to Accelerate PARAFAC", EUSIPCO 2005, September 4-8, Antalya, Turkey.

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